Committed Credit Lines

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Pricing and Liquidity Management

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Index

Introduction

- Loan commitments or credit lines are the most popular form of bank lending representing a high percent of all commercial and industrial loans by domestic banks.
- Loan commitments allow firms to borrow in the future at terms specified at the contract's inception.
- The model we propose simple, analytically tractable approach that incorporates the critical features of loan commitments observed in practice:
  - random interest rates;
  - multiple withdrawals by the debtor;
  - impacts on the cost of liquidity to back-up the withdrawals;
  - interaction between the probability of default and level of usage of the line.

Index

The model

- The bank has a portfolio of m different credit lines, each one with a given expiry T_i $(i = 1, 2, ..., m)$.
- Each credit line can be drawn within the limit L_i at any time t between today (time 0) and its expiry.
- We assume the there is a withdrawal intensity indicating which is the used percentage of the total amount of the line L_i at a given time t:
  - from each credit line i, the borrower can withdraw an integer percentage of its nominal: $1\%, 2\%, \ldots, 100\%$;
  - each withdrawal is modelled as a jump from a Poisson distribution (one specific to each credit line). This distribution can not have more than 100 jumps. As an example, a 3% withdrawal is equivalent for the Poisson process to jump 3 times;
  - the withdrawal intensity $\lambda_i(t)$ determines the probability of the jumps during the life of the credit line. This intensity is stochastic (so we have a doubly stochastic Poisson process).

Stochastic Intensity

- The stochastic intensity allows to model:
  - the documented correlation between the worsening of the creditworthiness of the debtor and the higher usage of the amount L_i of the credit line;
  - the correlation between the probabilities of default of the debtors of the m credit lines.
- Both effects have a heavy impact on the single and joint distributions of the usage of the credit lines.
- The liquidity management at a portfolio level is enhanced, since the bank can properly take into account the joint distribution
- The correlation between debtors determines in a more precise fashion the value of credit lines and the credit VaR, relying on a framework more **robust** and **consistent** than simple credit conversion factors.

Stochastic Intensity

The stochastic withdrawal intensity $\lambda_i(t)$, for the i-th debtor, is a combination of three terms:

$$\lambda_i(t) = \alpha_i(t)(\varrho_i(t) + \lambda_i^D(t))$$

- A multiplicative factor $\alpha_i(t)$ of a **deterministic** function of time $\varrho_i(t)$, which can be used to model the withdrawals of the credit line independent from the default probability of the debtor.
- A multiplicative factor $\alpha_i(t)$ of the **stochastic** default intensity $\lambda_i^D(t)$ (default is modelled as a rare event occurring with this intensity).
- We model the correlation between debtor by assuming that the default intensity is the sum of two separate components:

$$\lambda_i^D(t) = \lambda_i^I(t) + p_i \lambda^C(t)$$

Stochastic Intensity

The two components of the default intensity:

$$\lambda_i^D(t) = \lambda_i^I(t) + p_i \lambda^C(t)$$

are

- a **systematic** intensity $\lambda^C(t)$ is common to all m credit lines;
- an **idiosyncratic** intensity $\lambda_i^I(t)$ is specific for the i-th credit line;
- p_i is a constant between 0 and 1, and it commands the degree of correlation amongst the debtors.

All these processes are independent.
All the stochastic intensities follow a CIR distribution of the kind:

$$d\lambda_t = \kappa[\theta - \lambda_t]dt + \nu\sqrt{\lambda_t}dW_t$$

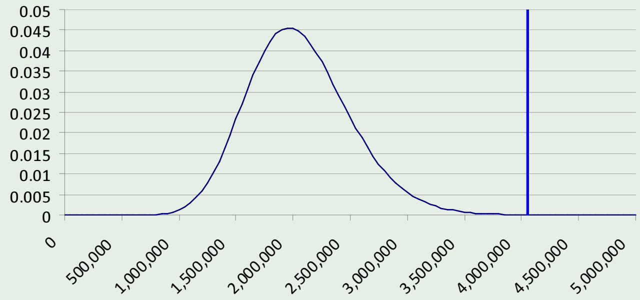
Index

Usage Distribution of a Single Credit Line

- We can use the above framework to compute the usage distribution of a single credit line: this can be useful also for pricing purposes.
- As an example, we assume the following set-up:
  1. The default intensity for the debtor is given by a combination of CIR processes with parameters:
     - $k^C = 0.8$, $\sigma^C = 20\%$, $\theta^C = 1.5\%$, $\lambda^C(0) = 2\%$.
     - $k_1^I = 0.8$, $\sigma_1^I = 15\%$, $\theta_1^I = 2\%$, $\lambda_1^I(0) = 1.5\%$.

     and $p_i = 0.25$. This means that the probability of default is about 2% in 1 year, and it declines on average in the future toward a long term average of about 2%.
  2. The deterministic intensity ϱ is constant and equal to 2%.
  3. The multiplying factor is also constant and equal to $\alpha = 1000$. Given this, we have an average withdrawal intensity at time 0 equal to $1000 \times (2\% + 2\%) = 40$ or 40% of the total amount of the line (since each jump corresponds to 1% of usage).
  4. The credit line nominal is equal to 5 mln.
  5. The end of the line is in 1 year.

Usage Distribution of a Single Credit Line



The figure shows the usage distribution of the credit line over a period of 1 year. The total amount of the credit line is Euro 5 mln. The average usage is 2,058,433. We added the 99%-percentile of this distribution which has the following value: 4,050,000.

Joint Usage Distribution of a Several Credit Line

- In a more general setting, we have to consider a portfolio of *m* different credit lines.
- We need to compute a joint usage distribution of all the credit lines and this is possible since we know the distributions of the single lines. It is not a simple sum, but mathematically speaking we operate a *convolution* of all the distributions.
- We show a simple example of two credit lines:
  - They have nominal amount equal to 7 mln and 5 mln, respectively. Each jump size is then 7 mln $\times 1\% = 70,000$ and 5 mln $\times 1\% = 50,000$.
  - The whole portfolio may reach 12 mln maximum total exposure to the two debtors.
  - All possible combinations of this two sizes up to 12 mln create all the entire portfolio possible withdrawals: 0, $50,000$, $70,000$, $100,000$, ...
- The approach can be easily extended to a general case.

Joint Usage Distribution of a Several Credit Line

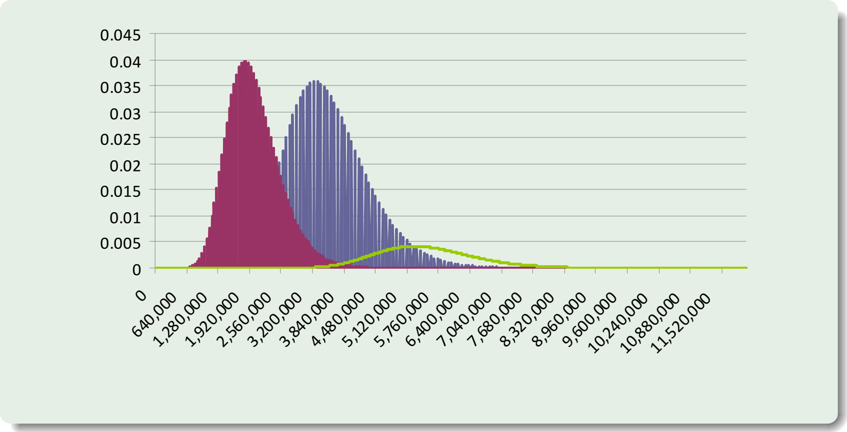
- The withdrawal intensity is given by the following parameters:

$$\lambda_1(t) = 2\% + \lambda_1^I(t) + 40\%\lambda^C(t), \quad (1)$$

$$\lambda_2(t) = 2\% + \lambda_2^I(t) + 40\%\lambda^C(t) \quad (2)$$

  where
  - $k^C = 0.8$, $\sigma^C = 20\%$, $\theta^C = 1.5\%$, $\lambda^C(0) = 1.5\%$.
  - $k_1^I = 0.8$, $\sigma_1^I = 15\%$, $\theta_1^I = 2\%$, $\lambda_1^I(0) = 1\%$.
  - $k_2^I = 0.8$, $\sigma_2^I = 2\%$, $\theta_2^I = 4\%$, $\lambda_2^I(0) = 0.5\%$.
- The multiplying coefficient for the first intensity is $\alpha_1 = 1139$ in order to have an expected usage near 50% ($E_1 = 50.0074$).
- For the second, we chose $\alpha_2 = 623$ in order to have an expected usage of 40% ($E_2 = 39.0421$).
- The expiry of the line is 1 year.

Joint Usage Distribution of a Several Credit Line



The figure presents the usage distributions for the two credit lines (in blue and in red lines). The joint distribution of the whole portfolio is also shown (in green line).

Joint Usage Distribution of a Several Credit Line

- The expectations for the two credit lines' usage distributions, on money-basis, are:
  - $E_1 \cong 3.49$ mln;
  - $E_2 \cong 2.0$ mln;
- The expected usage for the entire portfolio is $e_{tot} = 5.5$ mln, which is the sum of the two lines considered separately.
- Although the cumulated expected usage is not affected by the correlation between two debtors, yet the entire usage distribution is affected.
- More specifically, tails are heavily affected by the correlation between debtors, so that the portfolio liquidity management cannot disregard it.
- The plots of single portfolio seem to be higher. This is due to the fact that they do not have to be spread along the whole grid, but just in those points which are multiples of Euro 50,000 and Euro 70,000, respectively (the jump size for each of the two lines).

Joint Usage Distribution of a Several Credit Line

- In the present framework, the correlation amongst different debtors play a great role in determining the joint usage distribution.
- The p_i's coefficient weights the dependence of the default intensity on the common factor, and then it is an indication of the correlation amongst debtors.
- The higher the correlation (i.e.: p_i's), the larger the expected usage and the 99%-percentile unexpected usage.
- Even if for different scenarios (created by different p_is), the expected usage is equal for the single credit lines, the expected joint usage and the percentile changes with the choice of p_i.

Joint Usage Distribution of a Several Credit Line

We choose 3 credit lines, 5 mln, each and two opposite scenarios:

- **First Scenario**: we choose $p_i = 0.999$ and calibrate and the parameters of the default intensity so as to have an expected usage of 50%:
  - $\alpha = 1250$
  - $k^C = 0.8$, $\sigma^C = 20\%$, $\theta^C = 1.5\%$, $\lambda^C(0) = 2\%$.
  - $k_1^I = 0.8$, $\sigma_1^I = 15\%$, $\theta_1^I = 2\%$, $\lambda_1^I(0) = 1.99\%$.
- **Second Scenario**: we choose $p_i = 0.001$ and calibrate again α and the parameters of the default intensity so as to have an expected usage of 50%:
  - $\alpha = 1134$
  - $k^C = 0.8$, $\sigma^C = 20\%$, $\theta^C = 1.5\%$, $\lambda^C(0) = 2\%$.
  - $k_1^I = 0.8$, $\sigma_1^I = 15\%$, $\theta_1^I = 2\%$, $\lambda_1^I(0) = 0.002\%$.
- In both cases the expected usage is almost 7.5 mln.
- The 99%-percentile usage is 14,750,000 in the first scenario and $9,150,000$ in the second scenario.

Joint Usage Distribution of a Several Credit Line

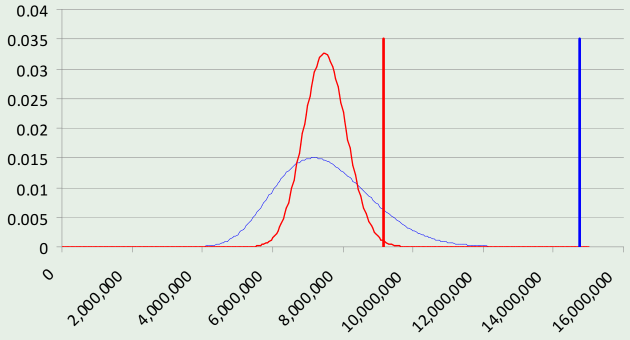


Figure shows the joint usage distribution, and the 99% precentile, for first scenario (in red) and for the second scenario (in blue). It is quite evident how, even if both scenarios have an expectation around 50%, the final percentile is quite different and it is higher when p_i is greater.

Index

Spread Options Sold by the Bank

- When a bank opens a credit line to a client, it allows to withdraw money within the line's limit, paying a predefined interest rate.
- The interest rate can be a fixed rate or, more frequently, a spread over a reference (risk-free) rate, *e.g.*: the Euribor.
- In either cases the bank is actually selling an option on the spread of the borrower, which clearly may have access to credit at future dates at a fixed spread until the expiry of the line.
- The credit line may be viewed as a portfolio of defaultable put options sold to the borrower on the borrower's own debt.
- The borrower is usually charged a stand-by fee, however, for the unused portion of the credit line, say F per unit of unused notional.
- Our objective is to calculate the market value to the borrower of the credit line, including the effect of any fees paid.
- We will also assume that the bank is default free.

Spread Options Sold by the Bank

- For each unit of notional, the option to use the line at time t actually means the option to sell, at a price equal to its notional value, a bond promising to pay the bank, at time $t + m$, the borrowed amount plus the interest, that is the reference index plus a contract spread $\bar{s}$.
- The market value of this obligation at time t, assuming that the borrower has survived to time t, is therefore a function of the difference between the spread of the borrower in t, $s(t)$, and the contract spread $\bar{s}$.
- The higher the market spread $s(t)$ with respect to the contract spread $\bar{s}$, the higher the value of the option to the borrower, at time t.
- The spread is a function of the probability of default of the borrower and the expected loss given the default.
- A very good approximation of the spread, assuming a loss given default as a percentage L of the market value of the withdrawn line, is: $s(t) = \lambda^D L$.
- The framework is rich enough to allow for a consistent pricing of the stand-by fee, at least for the factors due to the withdrawals and to the changing of the debtor's creditworthiness.

Spread Options Sold by the Bank: an Example

The bank opened a credit line for one year to a borrower.

The market rates at time $t = 0$ imply a term structure of zero rates and discount factors as shown beside.

For pricing purposes, we assume that the borrower withdraws at the start of each month, so that it has 11 spread options, the last one expiring at the end of the 11^{th} month. The other data referring to the credit line are also shown beside below.

Month	Zero Rate	Disc. Fact
1	1.25	0.998959
2	1.30	0.997836
3	1.37	0.996581
4	1.44	0.995212
5	1.51	0.993728
6	1.58	0.992131
7	1.65	0.990421
8	1.72	0.988599
9	1.79	0.986665
10	1.86	0.984620
11	1.93	0.982464
12	2.00	0.980199

α	1000
ρ	2%
$\bar{s}$	1%
LGD	50%

Spread Options Sold by the Bank: an Example

The fair value of the fee F to pay monthly on the unused part of the line is shown for different levels of the intensity of default λ^D at the start of the contract.

It is the level of the fee that makes nil the value of the contract at the inception, considering the optionality sold by the bank, the probability of withdrawals and the expected losses on the default of the borrower.

The fee is paid only if there is an unused part of the line and if the borrower has not defaulted yet.

λ_0	1.00%	κ	0.8
θ	2.00%	σ	40%
Stand-by Fee		0.012%	

λ_0	3.00%	κ	0.8
θ	2.00%	σ	40%
Stand-by Fee		0.081%	

λ_0	5.00%	κ	0.8
θ	8.00%	σ	40%
Stand-by Fee		0.421%	