Credit VaR: Pillar II Adjustments

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Hypothesis of the Model

The Basel 2 Capital Accord, Pillar I, is based on the so called "Asymptotic Single-Risk Factor" (ASRF) model, originally introduced by Vasicek in 1991. The value at risk is calculated on a *limiting* portfolio loss distribution, obtained under the following assumptions:

1. A unique systematic risk factor (single factor model).
2. An infinitely granular portfolio *i.e.* characterized by a large number of small size loans.
3. Dependence structure among different obligors described by the gaussian copula.

Credit VaR, IRB: Formula and *Input*

$$VaR^{BI}_{99.9\%} = 1.06 \sum_{i=1}^{M} EAD_i LGD_i \, N\left(\frac{N^{-1}(PD_i) + r_i \, N^{-1}(0.999)}{\sqrt{1 - r_i^2}} \right) MA(PD_i, \mathcal{M}_i)$$

- PD_i = probability of default of obligor i,
- LGD_i = loss given default,
- EAD_i = exposure,
- $\mathcal{M}_i$ = effective maturity of loan i,
- r_i =factor loading of the systematic risk factor,

$$r_i^2 = 0.12 \times (1 + e^{(-50 \times PD_i)})$$

- MA = maturity adjustment,
- $N(.)$ = normal cumulative distribution function,
  $N^{-1}(.)$ = inverse of the normal cumulative distribution function.

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Inclusion of further risks

Violations of the hypothesis underlying the ASRF model give rise to corrections which are explicitly taken into account by the BCBS under the generic name of **concentration** risk. They can be classified in the following way:

1. **Name concentration**: "*imperfect diversification of idiosyncratic risk*", i.e. imperfect granularity in the exposures
2. **Sector concentration**: "*imperfect diversification across systematic components of risk*"
3. **Contagion**: "*exposures to independent obligors that exhibit default dependencies, which exceed what one should expect on the basis of their sector affiliations*"

From a practical point of view, such contributions require further *inputs:*

$$\{PD_i, LGD_i, EAD_i, \mathcal{M}_i\} \quad + \quad \text{correlations}$$

Requirements of a General Model

A model, apt to evaluate consistently all kinds of concentration risk in the regulatory framework, needs to satisfy the following requirements:

- **Smooth connection** between the Pillar I VaR and the total VaR: ideally, the model should start from the same analytical formula derived in the Pillar I case and corrections should be added, accordingly to the regulation.
- **Consistency** of the total result: most credit VaR models produce total VaR estimates, that, in the majority of cases, are lower than the Pillar I VaR. If on one hand, the calculation of a VaR comprehensive of all risks seems therefore redundant, on the other, this disagrees explicitly with the request of the Regulator, according to which the Pillar I result underestimates the "true" credit VaR.
- **Intuitiveness** of input data: these should be ready available in the banking system and easily identifiable.

The Model

The model we propose has the following strengths:

- Credit VaR is calculated **analytically**, with huge saving of computational time.
- **Consistency** with the regulatory model is perfectly met, for two reasons:
  - The starting point is the same as in the Pillar I case.
  - The calibration procedure to the "*benchmark*" portfolio makes the single-factor formulation of the Regulator comparable with the multi-factor setup of the model.
- **Significance** of the adjustments, whose contribution is additive with respect to the Pillar I result, unless an important diversification effect among sectors occurs in the bank portfolio.

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Granularity Risk I

- The granularity adjustment is calculated through a second order Taylor expansion of the "true" VaR with respect to VaR^{ASRF}, computed on the asymptotic loss distribution which satisfies the hypothesis of the ASRF model.
- Such a term vanishes, for a sufficiently large number of obligors.
- For instance, Bank of Italy suggests a granularity adjustment proportional to the Hirschmann-Herfindahl index

$$HHI = \frac{\sum_{i=1}^{M} (EAD_i)^2}{\sum_{i=1}^{M} EAD_i^2}.$$

The proportionality constant $C(\rho, PD, LGD)$ is estimated through calibration.

Granularity Risk II

We propose a closed form solution, which bypasses calibration

$$\Delta VaR_q^{GA} \equiv VaR_q - VaR_q^{ASRF} = f(PD, LGD, \{\rho\})$$

In general, the outcome is an involved function of the model parameters (*PD*, *LGD* and correlations), which becomes simply proportional to the *HHI* index for homogeneous parameters.

Sector Risk

- Several systematic risk factors can lead to an imperfect diversification.
- We consider N sectors of industry-geographic type.
- In addition to the inputs of the ASRF model, we need to feed the following external data:
  - obligor/sector correlations,
  - sector/sector correlations.
- The effect of sector risk on VaR crucially depends on the distribution of portfolio loans on sectors.
- In order to compare the multi-factor VaR with the Pillar I VaR (obtained for a single-factor portfolio which mimics the effect of a "well diversified" multi-factor one), it is necessary to identify a benchmark portfolio with which to calibrate the model VaR on the regulatory one.

Benchmark Portfolio and Calibration

The reference portfolio must have the following features:

- homogeneity in the exposures,
- "good diversification" among sectors. For instance,

	A*	B	C1	C2	C3	D	E	F	H	I	J
Italy	5%	8%	16%	32%	5%	17%	7%	2%	1%	1%	6%
Germany	1%	6%	12%	33%	7%	15%	6%	9%	3%	1%	7%

* GICS classification scheme for sector activities

Assuming the systematic factor loadings r_i have an expression similar to the one proposed by the Regulator, e.g. in the Italian case ($\alpha = 0.12$ for BI)

$$r_i^2 = \alpha \left(1 + e^{-50PD_i}\right)$$

we retrieve the parameter α, by calibrating the model VaR on VaR^{BI}.
(The result can be easily modified in order to take into consideration the VaR proposed by BCBS.)

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Contagion Risk I

Contagion risk accounts for interaction effects among different obligors (of business type, legal, financial...).

We propose a contagion specification which takes into account

- the *overall* sensitivity of each obligor to contagion, through the contagion factor loading g_i,
- the dependence of each obligor on specific groups of contagion agents. In order to define this more precisely, we divide obligors into two classes:
    - "I-firms": obligors who are immune from contagion, but can propagate it;
    - "C-firms": obligors who can be infected.

  Each sector is divided into an "I"segment and a "C"one. The dependence of every "C"obligor on the "I"part of each sector is encoded into proper weights, which we propose to express in terms of revenues data.

Contagion Risk II

The new required inputs are:

- the discretional factor loading g_i,
- the weights expressing correlations of the type obligor/ "I" segment of each sector

$$\gamma_{ik} = \mathcal{C}\,\frac{R^I_{ik}}{R^I_i}\,,$$

$i = 1 \ldots M$ obligor, $k = 1 \ldots N$ sector,

$\mathcal{C}$ = normalization constant $= 1/\sqrt{\sum_{k=1}^{N}\left(\frac{R^I_{ik}}{R^I_i}\right)^2}$

$\{R\}$ = revenues

Contagion Risk III

Result:

$$\Delta VaR_q^C \equiv VaR_q^C - VaR_q^{ASRF} = VaR_q^{GA,C} + VaR_q^{\infty,C} \,,$$

where:

- $VaR_q^{ASRF} = f_1(PD, LGD, r)$ asymptotic component: it is not affected by contagion and it is based on a single "efficient" systematic risk factor, which takes into account the combined effect due to all sectors,
- $VaR_q^{GA,C} = f_2(PD, LGD, \rho^C)$ granularity adjustment in the presence of contagion,
- $VaR_q^{\infty,C} = f_3(PD, LGD, \rho^C)$ multi sector correction in the presence of contagion.

$\rho^C = \rho^C(PD, LGD, r, g, \gamma_{ik},$ sector correlations)

(If $g = 0$ contagion is switched off.)

Model Specification

The model allows to define, without any kind of constraint, the following parameters:

1. Number of rating classes: each of them will be characterized by a *PD* and an *LGD*.
2. Number of sectors: they can be defined according to economic and/or territorial criteria.
3. Number of correlation *buckets* between obligors and sectors.
4. Obligors assignment to one or more sectors.
5. Obligors assignment to the "I" class or the "C" class.
6. Definition of the weights through which infected obligors can depend on one or more infecting segments.

Numerical examples

- $M = 5000$, $N = 11$, 7 rating classes, 3 correlation buckets, $g = 0.5$, $H =$ homogeneous portfolio, $C =$ concentrated portfolio, $VaR^{BI} = 5.75\%$:

	$VaR^{ASRF}_{99.9\%}$	$VaR_{99.9\%}$	$\Delta^{(*)}_{tot}$	Δ_C	Δ_{GA}	Δ_∞	t
bmk	5.62%	6.33%	0.69%	0.56%	0.04%	0.65%	1012
H	5.28%	5.98%	0.70%	0.60%	0.04%	0.66%	756
C	7.72%	8.38%	0.66%	0.62%	0.04%	0.62%	641

- $M = 500$, $N = 11$, 7 rating classes, 3 correlation buckets, $g = 0.5$, $H =$ homogeneous portfolio, $C =$ concentrated portfolio, $VaR^{BI} = 5.80\%$:

	$VaR^{ASRF}_{99.9\%}$	$VaR_{99.9\%}$	Δ_{tot}	Δ_C	Δ_{GA}	Δ_∞	t
bmk	5.25%	6.36%	1.10%	0.56%	0.48%	0.62%	484
H	4.95%	6.05%	1.10%	0.60%	0.45%	0.64%	503
C	7.10%	8.13%	1.03%	0.62%	0.41%	0.62%	270

$(*)\ \Delta_{tot} = \Delta_{GA} + \Delta_\infty$

Comparison with CreditMetrics and CreditRisk+ I

We compare our model with:

- a restricted version of CreditMetrics (CM2S), which only considers default events,
- a version of CreditRisk+ (CR+), characterized by a single-factor and a discretional parameter, σ.

(Data are extracted from Gordy 2000.)

Comparison with CreditMetrics and CreditRisk+ II

- Portfolio homogeneous in the exposures, $N = 1$, 7 rating classes, no contagion. $M = 400$ e $M_{Gordy} = 5000$.

	Model	CM2S	CR+ $\sigma = 1.5$
$q = 99.97\%$	0.0264	0.02649	0.03187
$q = 99.5\%$	0.0174	0.01747	0.02009
$q = 99\%$	0.0153	0.01527	0.01728

- Portfolio granular in the exposures, $N = 1$, 7 rating classes, no contagion. $M = 400$ e $M_{Gordy} = 5000$.

	Model	CM2S	CR+ $\sigma = 1$	CR+ $\sigma = 1.5$	CR+ $\sigma = 4$
$q = 99.97\%$	0.0341	0.02714	0.02736	0.03225	0.05149
$q = 99.5\%$	0.0226	0.01795	0.01847	0.02033	0.02488
$q = 99\%$	0.0199	0.01578	0.01628	0.01749	0.01916

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