# On the Dynamic Replication of the DVA: Do Banks Hedge their Debit Value Adjustment or their Destroying Value Adjustment?*

Antonio Castagna

iason

First Version: December 1, 2011
This version: July 11, 2012

## 1 Introduction

We investigate if it is feasible to dynamically replicate the **DVA**. If the answer is positive, the **DVA** is a quantity that can be fairly deducted from the liabilities of a financial institution. In this case, the argument in Burgard and Kjaer [5], where a dynamic replication strategy is derived in details, will be accepted. If on the contrary the answer is negative, then the **DVA** should be considered as a cost and as such it should be deducted from the equity of the financial institution. In this second case we confirm the results in Castagna [6].

We will analyse the problem under a very wide perspective. We will show that dynamically replicating the **DVA** hides very subtle assumptions about the composition of the balance sheet of the financial institution. We will also point out which are the (negative) consequences for the business if the financial institution will organize the derivatives business so as to hedge and replicate also the **DVA** and we will demonstrate how the bank's franchise will be gradually eroded.

## 2 The Gain Process

We start from a very basic concept in Option Pricing Theory. It seems that we are just repeating very well-known results, but we do that because we do not understand why these results are strangely forgotten when the **DVA** is involved in the analysis.

---

*Iason ltd. Email: antonio.castagna@iasonltd.com. This is a preliminary and incomplete version. Comments are welcome.

Let $X_t$ be the stochastic variable representing the price of an asset. The evolution of $X_t$ is given by the following SDE (Ito process):[1]

$$dX_t = \mu(X,t)dt + \sigma(X,t)dZ_t \tag{1}$$

Define a trading strategy as an adapted process $\theta$ specifying at each state $\omega$ and time $t$ the number $\theta_t(\omega)$ of units of the asset held by an economic operator. The gain process generated by $\theta$ is the stochastic integral:

$$\int_0^T \theta_t dX_t = \int_0^T \theta_t \mu(X,t)dt + \int_0^T \theta_t \sigma(X,t)dZ_t \tag{2}$$

Basically the gain process indicates the gains (considered both positive and negative) generated by the $\theta_t$ units held at each time $t$ given the variation $dX_t$ of the asset.

Assume we have a constant quantity $\overline{\theta}$ held between time $T$ and $T'$. The gain process is simply $\overline{\theta} \int_T^{T'} dX_t = \overline{\theta}[X_{T'} - X_T]$. It is immediate to check that the gain process is nil with probability 1 between the two times $T$ and $T'$ if $\overline{\theta} = 0$.

Assume now we short 1 unity of the asset between 0 and $T$, and that we buy it back between $T$ and $T'$, so that $\theta_t = -1$ for $t \in \{0, T\}$, and $\theta_t = 0$ for $t \in \{T, T'\}$. The total gain process is:

$$-1 \times [X_T - X_0] + 0 \times [X_{T'} - X_T] = -1 \times [X_T - X_0]$$

Calculations are quite simple and we are trivially stating that when we do not hold any quantity if the asset for a given period we do not earn any gain.

The asset can be a stock, a commodity or a bond. When a financial institution, say a bank, issues at time 0 a bond ($X = B$) to finance its business activity, this is as it goes short (*i.e.*: sells without having previously bought) the same bond ($\theta_0 = -1$). Assume for simplicity, but with no loss of generality, that the bond is a zero-coupon. The bond is usually paid back at the expiry $T$ by the bank, which is in practice buying back the bond shorted ($\theta_T = 0$), and the gain process typically entails a loss for the issuer ($-1 \times [B_T - B_0]$) which is the amount of interests granted to the bondholder. The bond can be bought back also before the expiry, at time $u < T$, thus producing a gain $-1 \times [B_u - B_0]$ that could be positive or negative if we are in a stochastic interest rate and default probability economy. On the other hand, if we are in a deterministic interest rate and default probability economy, buying back issued bonds always implies a loss for the bank, although lower than that suffered at the expiry (*i.e.*: the bank pays fewer interests, since it keeps a short position in the bond for a shorter period). It is clear that from the time $u$, until the expiry $T$, the gain process is nil since $\theta_t = 0$ for $t \in \{u, T\}$, unless the bank decides to issue once again the same bond.

# 3 Dynamic Replication of a Defaultable Claim

Dynamic replication relies on the ability to attain the same pay-off structure of a derivative contract via a trading strategy in primary securities (*e.g.*: stocks and bonds). Assume we

[1] For technical details see, for example, Duffie [7].

have a vector of $N$ securities defined by the price process $X = (X^1, ..., X^N)$. We want to replicate dynamically a derivative claim whose terminal pay-off at the expiry $T$ is $V_T$ and whose initial price a time 0 is $V_0$.

The replication portfolio is set up in 0. We have to find a trading strategy $\theta$, such that it satisfies the following well-known conditions:[2]

1. Self financing condition, that is: no other investment is required in operating the strategy besides the initial one:

$$\theta_t \cdot X_t = \theta_0 \cdot X_0 + \int_0^t \theta_s dX_s \tag{3}$$

2. Replicating condition, that is: at any time $t$ the replicating portfolio's value equals the value of the contract and of the collateral account:

$$V_t = \theta_t \cdot X_t \tag{4}$$

for $t \in [0, T]$.

We apply the replication to a defualtable derivative contract $\widehat{V}$, whose corresponding default-risk free value is denoted with $V$: the deal is written between the bank $B$ and a counterparty $C$. In building the replication portfolio we strictly follow Brugard and Kjaer [5], whom we refer to for details. $\widehat{V}$ is the value of the contract seen from the counterparty $C$ perspective, and it is also the value that the bank $B$ has to replicate once it closes the deal, so as to hedge the exposure it has towards $C$.

Let $X^1 \equiv S$ be the underlying asset which the contract's pay-off is contingent upon, $X^2 \equiv P$ be a risk-free zero-coupon bond, $X^3 \equiv P^B$ be a default-risky zero-coupon bond issued by the bank $B$ and finally $X^4 \equiv P^C$ a default-risky zero-coupon bond issued by the counterparty $C$. The two risky bonds depend each on the default of the respective issuer, so that they can be used to hedge the exposures that the derivative contract implies to the parties' defaults. Both bonds have zero recovery when the issuer's default occurs.

Dynamics for $S$ is the same as in (1), whereas the dynamics for the bonds are:

$$dP_t = r_t P_t dt$$

$$dP_t^B = (r_t + \lambda_t^B) P_t^B dt - dJ^B P_t^B$$

$$dP_t^C = (r_t + \lambda_t^C) P_t^C dt - dJ^B P_t^C$$

where $r_t$ is the deterministic time dependent instantaneous risk-free interest rate and $\lambda^I$ is the yield spread of the operator $I \in \{B, C\}$, which in equilibrium should be also the instantaneous default intensity.

We apply Ito's lemma to the value function $\widehat{V}(S, t, J^B, J^C)$, where $J^B$ and $J^C$ are two point processes that jump from 0 to 1 on default of, respectively, $B$ and $C$ with default intensity $\lambda^B$ and $\lambda^C$. We get:

$$d\widehat{V}_t = \mathcal{L}^a \widehat{V}_t dt + \sigma(t, S) \frac{\partial V_t}{\partial S_t} dZ_t + D\widehat{V}_t^B dJ^B + D\widehat{V}_t^C dJ^C \tag{5}$$

---

[2]For a general treatment see Duffie [7] or Merton [8].

where we used the operator $\mathcal{L}^a\cdot$ defined as:

$$\mathcal{L}^a\cdot = \frac{\partial\cdot}{\partial t} + a(S,t)\frac{\partial\cdot}{\partial S_t} + \frac{1}{2}\sigma^2(S,t)\frac{\partial^2\cdot}{\partial S_t^2} \tag{6}$$

and we set the variation of the value of the contingent claim upon the default of one of the two parties as:

$$D\widehat{V}_t^B = \widehat{V}(S,t,1,0) - \widehat{V}(S,t,0,0)$$

and

$$D\widehat{V}_t^C = \widehat{V}(S,t,0,1) - \widehat{V}(S,t,0,0)$$

On the other hand, bank $B$ wants to build a replicating portfolio comprising a quantity $\Delta_t$ of underlying asset, $\alpha_t^B$ of zero-coupon bonds issued by the bank itself, $\alpha_t^C$ of zero-coupon bonds issued by the counterparty $C$, and finally an amount of cash $\beta_t$, so that it satisfies the two conditions stated above:

$$\widehat{V}_t = \Delta_t S_t + \alpha_t^B P_t^B + \alpha_t^C P_t^C + \beta_t \tag{7}$$

and

$$d\widehat{V}_t = \Delta_t dS_t + \alpha_t^B dP_t^B + \alpha_t^C dP_t^C + d\beta_t \tag{8}$$

The quantity $\beta_t$ is defined as:

$$\beta_t = (\widehat{V}_t - \delta_t S_t - \alpha_t^B P_t^B - \alpha_t^C P_t^C)$$

and its evolution depends on the assumptions made on how to finance the asset and the two bonds.[3] We assume that the position in the underlying asset is financed by a repo transaction: if the repo rate is $r^R$ and the asset grants a continuous the yield $y$, then for this part cash will evolve as:

$$\Delta_t(y_t - r_t)S_t dt$$

The position in the counterparty's bonds can be financed at repo as well, and we assume zero haircut and a repo rate equal to the risk free rate $r$, so that the evolution for this component of the cash is:

$$-\alpha_t^C r_t P_t^C dt$$

Finally, the position in bank's bonds is financed by the amount $\widehat{V}$ that the replication strategy implies as an investment at the start of the contract; any remaining sum of cash will be invested at the risk free rate if positive, or financed at the risk free rate plus the bank's funding spread $s^B$, thus yielding the dynamic for this last part of the cash:

$$r_t(\widehat{V}_t - \alpha_t^B P_t^B) + s_t^B(\widehat{V}_t - \alpha_t^B P_t^B)^-$$

Collecting the results we eventually get:

$$d\beta_t = [\Delta_t(y_t - r_t)S_t dt - \alpha_t^C r_t P^C dt + r_t(\widehat{V}_t - \alpha_t^B P_t^B) + s_t^B(\widehat{V}_t - \alpha_t^B P_t^B)^-]dt$$

[3] We accept the same assumptions as in Burgard and Kjaer [5]. We are not affirming that we agree on all of them, but they are immaterial to the point we want to prove, so we do not try to introduce other assumptions that we may deem more reasonable.

By equating (8) with (5) we can derive the quantities of the different assets to include in the portfolio so that it perfectly replicates the derivative contract. It can be shown that they are:

$$\delta_t = \frac{\partial V_t}{\partial S_t}$$

$$\alpha_t^B = -\frac{\Delta \widehat{V}_t^B}{P_t^B} = \frac{\widehat{V}_t^B - (M^- + R^B M^+)}{P_t^B}$$

$$\alpha_t^C = -\frac{\Delta \widehat{V}_t^C}{P_t^C} = \frac{\widehat{V}_t^C - (M^+ + R^C M^-)}{P_t^C}$$

where we have defined $M$ as the mark-to-market value of the contract upon default of one of the two parties, and with $R^I$, $I \in \{B, C\}$, the recovery fraction of the contract paid by defaulting party $I$ to the other one.

By re-arranging terms, it can be shown that the final PDE, whose solution is the value of the derivative contract, is:

$$\mathcal{L}^{r-y}\widehat{V}_t dt = r_t \widehat{V}_t dt + s_t^B M^- - \lambda_t^B (M^- + R^B M^+) - \lambda_t^C (M^+ + R^C M^-) \tag{9}$$

If we assume that the mark-to-market value on the default of one of the parties is the defaultable value of the contract, then $M = \widehat{V}$. [4] In this case the total value of the contract can be decomposed as $\widehat{V} = V + \widehat{U}$, that is a risk free component plus an adjustment due to credit events, which is the solution of the PDE:

$$\mathcal{L}^{r-y}\widehat{U}_t dt = s_t^B (V_t + \widehat{U}_t)^- + (1 - R^B)\lambda_t^B (V_t + \widehat{U}_t)^+ + (1 - R^C)\lambda_t^C (V_t + \widehat{U}_t)^-$$

where $V$ is known and can be derived by standard techniques. Application of the Feynman-Kac theorem provides the solution of the adjustment term as:[5]

$$\begin{aligned}
\widehat{U}(S,t) = &- (1 - R^B) \int_t^T \lambda_s^B e^{-\int_s^T r_u du} \mathbf{E}[(V(S,s) + \widehat{U}(S,s))^+] ds \\
&- (1 - R^C) \int_t^T \lambda_s^C e^{-\int_s^T r_u du} \mathbf{E}[(V(S,s) + \widehat{U}(S,s))^-] ds \\
&- \int_t^T s_s^B e^{-\int_s^T r_u du} \mathbf{E}[(V(S,s) + \widehat{U}(S,s))^-] ds
\end{aligned} \tag{10}$$

Formula (10) contains two elements referring to counterparty credit risk: the first line, on the right hand side, shows the **CVA** (from $C$'s perspective) and it is the correction

[4] Burgard and Kjaer [5] examine also the case when the mark-to-market on default is the risk-free value of the contract $M = V$. The analysis we show applies exactly also to this second case. For an analysis of the close-out conventions upon default, see Brigo and Morini [4].

[5] A similar pricing formula, accounting for bilateral counterparty risk adjustment but not for funding rate, is in Brigo and Capponi [3], although it is not derived by a dynamic replication argument. For a more general formula including also collateralization and funding costs, see Pallavicini et al. [10].

to the risk free fair value $V$ to remunerate the risk $C$ bears for $B$'s default (recall that the value $V$ is as of seen from $C$ perspective); the second line is the **DVA** (again, from $C$'s perspective), and it is the correction to remunerate the specular risk that $B$ bears for $C$'s default. Finally, the third line, shows a cost that bank $B$ has to bear when trying to replicate a long position in the contract and it is related to the funding spread it pays over the risk free rate.

Since we are interested in studying the effectiveness of the replication strategy of $U(S,t)$ from the bank $B$'s perspective, and more specifically of its Debit Value Adjustment, we will focus on the **DVA** for the bank's perspective, *i.e.*: to the quantity in the first line of (10); specularly, what is the **DVA** from the counterparty $C$'s perspective (second line in (10)), is in fact the **CVA** for the bank.

Let us have a closer look at which is the sign of the quantities that the bank has to hold in the portfolio to replicate $U(S,t)$ and hence to hedge it specular position $-U(S,t)$ against counterparty $C$.[6] The **CVA** (or **DVA** from counterparty's perspective, the second line in (10)) can be quite easily replicated by a selling an amount of bonds issued by counterparty $C$ equal to $\alpha_t^C = (1-R^C)\widehat{V}_t^-/P_t^C$: this can be achieved in a simple way by a repo agreement with a third party, or even in more "heterodox" ways such as buying credit protections via a CDS (with all the related *caveats*). The funding component of $U(S,t)$ (the third line) also is not very worrisome in terms of replicability: the bank has to issue new bonds to fund, in case, any negative cash flow originated by the setting up of the replication portfolio. Provided that there are no liquidity issues in the market, borrowing money from other operators should be an ordinary matter.

The replication of the **DVA** (that is, the **CVA** from the counterparty's perspective, the first line (10)) is trickier: it entails for the bank going long a quantity of its own bonds equal to $\alpha_t^B = (1-R^B)\widehat{V}_t^+/P_t^B$. Now, while going short its own bonds is relatively straightforward for the bank, since in the end it amounts to borrowing more money from the market, going long its own bond is not possible, although this is often overlooked in the literature. Actually also Burgard and Kjaer [5] suggest an apparently simple way for the bank to go long its own bonds by buying back bonds issued in the past. Although in theory one cannot exclude the possibility that a bank has never issued bonds in the past, in practice the strategy is admittedly not very difficult to implement: in fact banks regularly issue debt and there are many bonds in the market to buy back. So, is buy-back a strategy to go long its own bonds for the bank? The answer is definitely not, and not just because we object that is quite hard to find an issued bond in the market that exactly matches the features of the bond $P^B$ entering in the replication strategy.

In fact, there is a difference between buying a security and being long it. We have stressed that in section 2, when we have somehow redundantly shown that buying back a short position makes, obviously, the net position nil for the dynamic replicator: if the replica prescribes a long position in a given security, the replicator should keep on buying until she is net long the security. But when a bank buys back its own bonds, it is simply reducing its short position or it is making it at the maximum equal to zero, not going long

[6] The component $V(S,t)$ is dynamically replicated according the well known principles of the standard Option Pricing Theory: this indicates to hold a position $\Delta_t$ of the underlying asset at each time $t$, partially financed by shorting bonds or with a repo transaction, as it is the case in the analysis we are conducting.

its own bonds, *i.e.*: adding a positive amount in its portfolio. The gain process for the bonds bought back stops (since the quantity of the "bond" asset $\theta = 0$, using the notation in section 2); from the time of the buy-back on, the gain process simply sticks to amount of profits or losses generated since the issuance of the bond and no other variation occurs. So the replication of the **DVA** (from bank's perspective) is simply not happening since there is no real contribution from the gain process of the bank's bonds. This statement is quite strong and hinges on the statement that is not possible for the bank to go long its own bonds in any fashion: it can subject to possible critiques that we try and devise and rebut in next section.

# 4 Objections to the Statement "No Long Position in One's Own Bonds is Possible"

As a first attack to statement, one may object that it is true that the bank never goes really long its own bonds, but if one considers the replication strategy as a closed system, then the bank is actually long the bond so that the replication is effective. This objection is basically based on "abstract concept of an abstract concept", in terms of the philosopher Emanuele Severino.[7] The long position of the bank in its bonds within the closed system "dynamic replication of its **DVA**" without considering the total net position of the bank within its balance sheet (*i.e.*: the total of its assets and liabilities), which is the total system which also the "dynamic replication of its **DVA**" sub-system belongs to, is an abstract concept: being long in the sub-system means that a short position is opened somewhere else in the total "balance sheet" system, so as to preserve the zero position in bank's own bonds at an aggregated level. So in the end the bank is long in the "dynamic replication of its **DVA**" sub-system and short somewhere else in the overall "balance sheet" system. Besides, assuming that the long position in bonds is actually existing and producing effects, is an abstract concept as well, since at an aggregated level the effects are just offsetting each other.

In the end the bank cannot attain an effective long position in its own bonds, although it is possible at a sub-system level to assume this position, albeit it has to be counter-balanced at a system level by an opposite position. This means that if in the "dynamic replication of its **DVA**" sub-system the replication of the **DVA** is formally attained, in practice at the "balance sheet" level the replication is simply paid by abating some assets or increasing some liabilities, so that the net total result is that the replication strategy ends up with a loss (or differently said, as a cost) for the bank. This is simply due to the strength of logic consequences and with this in mind we will be better equipped also to face another possible critique. [8]

The second, and somehow subtler, critique one may raise to the impossibility to have

[7] See Severino [11], chapter 9.

[8] In simpler and more heuristic terms, this has been also shown in Castagna [6]. Below we will investigate better how the cost originating from the **DVA** replication affects negatively the bank's franchise.

a long position in bonds is based on the "funding benefit" argument. This argument basically goes like this: if the bank has money to buy back its bonds issued in the past, then it gets a benefit in term of smaller funding costs it pays on outstanding debt. So, the critique is actually accepting the statement that it is not possible for the bank to go long its bonds, but it introduces the gains, or benefits, that bank obtains with a smaller outstanding debt. The funding benefits are mentioned in Burgard and Kjaer [5] and also in Morini and Prampolini [9], although in vague terms. To be fully honest, there seems to be a ground for this argument, so we have to investigate it further to ascertain its validity.

Granted for sake of argument that the funding benefit really exists,[9] it is not clear how it is related to the replication of the bank's **DVA** and how its variations actually track the variations of the **DVA**. Anyway, since we are aware there is some sensibleness in the argument, we try and make it more explicit in next section, so as to fully investigate it.

# 5 DVA Replication by the Funding Benefit

There is something that can be hardly denied: if a bank has some cash and it can buy back all or a part of the bonds issued in the past, then it reduces the amount of debt. This is not precisely a funding benefit, which in our opinion should be defined as a saving of funding costs given a certain amount of debt, but can be loosely though as of a benefit since the bank has to pay less interests, taken in absolute value. So we loosely define funding benefit as the deduction of paid interests' amount one can obtain by reducing the total outstanding debt, after the buy-back.

When the replication strategy of a contract, subject to bilateral counterparty risk, prescribes to buy back one's own bonds, since the strategy is self-financing, it is also generating the amount of cash needed to perform this, so in the end we are sure that, as for the **DVA** component (from $B$'s perspective), the bank has cash to buy back a quantity of its own bonds thus reducing its total outstanding debt. Is this amount of debt "missing" from the original total amount really contributing to replicating the **DVA**? To inquiry that we need to consider the general picture, that is: we need to look at the entire bank's balance sheet and how assets and liabilities are originated.

Let us commence with a very basic situation, when the bank starts its activities at time 0, with an amount of capital $E$, deposited in an account $D_1 = E$. We observe the bank's activity at discrete time intervals of length $T$. In 0 the bank also issues an amount $K$ of zero-coupon bonds $P^B$ with unit face value and expiring in $3T$. Adopting the same notation introduced above, the amount of cash raised by the bank is $Ke^{-(r+s^B)3T}$ (recall that $s^B$ is bank's funding spread); this cash is used to buy $K$ zero-coupon bonds $P^Y$, issued by a third party $Y$, with unit face value and expiring in $3T$. The third party has a funding spread $s^Y$, so that the present value of the bond is $Ke^{-(r+s^Y)3T}$; if both bonds have the same funding spread, $s^B = s^Y$, then the money raised by the bank is enough to buy the bond of the issuer $Y$.

[9] Castagna [6] shows that actually the funding benefit is only a badly posed concept, since given an amount of funding available, the bank is paying in any case its funding spread without any saving, or benefit.

For the moment we assume that the funding spread is due to some unspecified factors and we do not try to link it to the default risk, which in reality should be the first cause of its existence. We will very quickly explicitly consider the default risk in the following analysis, but for now we simply disregard it. If this is the case, we can then affirm that the bank is operating a very simple replication strategy for the asset $A_1$, with the opposite sign so as to hedge it, via the issuance of its own bonds.

The marked-to-the-market balance sheet of the bank at time 0 looks like:[10]

| **Assets** | **Liabilities** |
|---|---|
| $D_1 = E$ <br> $A_1 = Ke^{-(r+s^Y)3T}$ | $L_1 = Ke^{-(r+s^B)3T}$ <br> ——— <br> $E$ |

Now, assume one period $T$ elapses and that the bank closes a derivative contract. To make things more explicit and to avoid unnecessary complications (but in any case with no loss of generality), we suppose that the bank $B$ sells to counterparty $C$ an option on some underlying $S$ whose value to the latter is $\widehat{V} = V + \widehat{U}$ (this choice will allow to exclude from the analysis the **CVA** for the bank, which is zero for short options); clearly the option is worth to the former the same value with the opposite sign. Since it has a negative value to bank $B$, the option is a liability; on the other hand, the premium paid by $C$ increases the cash available to $B$ and it is deposited in a deposits $D_2$. The balance sheet will be then:

| **Assets** | **Liabilities** |
|---|---|
| $D_1 = Ee^{rT}$ <br> $D_2 = \widehat{V}$ <br> $A_1 = Ke^{-(r+s^Y)2T}$ | $L_1 = Ke^{-(r+s^B)2T}$ <br> $L_2 = \widehat{V}$ <br> ——— <br> $E$ <br> $II_1 = (Ee^{rT} - E)$ |

The assets and liabilities accrue interests and a net profit $(Ee^{rT} - E)$ is earned between 0 and $T$. The bank starts also immediately the dynamic (reverse) replication strategy: for simplicity's sake we focus only on the **DVA** (from the bank's perspective) part of the quantity $U(S,t)$ in (10) (*i.e.*: the **CVA** from the counterparty's perspective), without considering the $\Delta$-hedge with the underlying asset. We assume that the quantity $\alpha^B$ of the bank's bond to buy back is exactly equal to $K$, or the amount of the bond outstanding issued at time 0. Obviously to buy back the bond, the amount of available cash in $D_2$ is abated correspondingly so that the balance sheet reads as:

[10] We do not follow the result in Castagna [6] that would make us consider the **DVA** of a contract as a cost to be deducted from the equity, since we want here to investigate if the **DVA** is actually a replicable quantity. We hence suspend our judgment on how to consider it and each contract is written in the balance-sheet at its trading price.

| Assets | Liabilities |
|---|---|
| $D_1 = Ee^{rT}$<br>$D_2 = \widehat{V} - Ke^{-(r+s^B)2T}$<br>$A_1 = Ke^{-(r+s^Y)2T}$ | $L_1 = 0$<br>$L_2 = \widehat{V}$<br>———<br>$E$<br>$II_1 = (Ee^{rT} - E)$ |

Here we get to the heart of the matter: no bond appears amongst bank's liabilities, and it seems that they even declined. In reality liabilities did not decline, on the contrary they increased since the bond has been replaced by a short position in the option that is worth (negatively) even more. In any case, the bond issued counterbalancing the asset $A_1$ is no more there, and this is a fact: it seems like that the bank has a long position in the asset that does not need to be financed by cash, whose availability for the bank increased as it is manifest by the amount in the deposit $D_2$ that was not existent before. This is what can be named "funding benefit", as we suggested above, and that apparently makes possible to have assets in the balance sheet without paying (or by paying less) funding costs explicitly.

We already showed elsewhere (in the work cited above) that this saving is only an illusion. But we would like to check here if this apparent saving is anyway effective in the replication strategy. And actually it could be effective if a set of circumstances are true. In fact, we already stressed that when the bank buys back it own bonds is not really going long them, but it is simply making nil its former position, considering things at a balance sheet level. The gain process that is needed in the replication strategy is only formally and abstractly produced (in case it is so at a lower sub-system level, such as a trading desk), but in practice it stops at the instant of the buy-back (although the gain obtained up to this instant is immaterial to the replication strategy of the **DVA** for the bank) and it keeps being constant until a new short position is opened by issuing once again the bond.

On the other hand, we can now argue that the asset $A_1$ is no more hedged (*i.e.*: replicated with the opposite sign) since the issued bond has been bought back and we can also argue that another gain process actually starts being produced. Indeed, after one period more elapsed, the balance sheet reads as:

| Assets | Liabilities |
|---|---|
| $D_1 = Ee^{r2T}$<br>$D_2 = (\widehat{V} - Ke^{(r+s^B)2T})e^{rT}$<br>$A_1 = Ke^{-(r+s^Y)T}$ | $L_1 = 0$<br>$L_2 = \widehat{V}$<br>———<br>$E$<br>$II_1 = (Ee^{r2T} - E)$<br>$II_2 = (\widehat{V} - Ke^{(r+s^B)2T})(e^{rT} - 1)$<br>$P\&L = K[e^{-(r+s^Y)T} - e^{-(r+s^Y)2T}]$ |

The amount of capital deposited in $D_1$ is accruing interests $II_1$, so that related profits increase; interests $II_2$ accrued on the deposits $D_2$ generate other profits $((\widehat{V} - Ke^{(r+s^B)2T})(e^{rT} -$

1)). We assume also that the value of the option stays constant for a "lucky" set of circumstances, so that it does not contribute to the period Profits&Losses. Then we can observe that we are left in the balance sheet with a profit equal to $K[e^{-(r+s^Y)T} - e^{-(r+s^Y)2T}]$ that would not be generated if the the bond issued by the bank were not bought back. In fact, the issued bond would generate a perfectly counterbalancing loss $K[e^{-(r+s^B)T} - e^{-(r+s^B)2T}]$ (since $s^B = s^Y$) and the total effect in the balance sheet would be zero. On the other hand, and for the same reason of equal funding spreads, the profit appearing in this case is the same as the profit that the bank could earn if it had a "true" long position in its own bonds.[11] If the replication strategy indicates a quantity $\alpha_T^B = \Delta\widehat{V}_T / P_T^B = K$, then in equation (8) (recalling we are working in a discrete-time setting)

$$\alpha_T^B \Delta P_T^B = \alpha_T^B \Delta P_T^Y = K[e^{-(r+s^B)T} - e^{-(r+s^B)2T}] = K[e^{-(r+s^Y)T} - e^{-(r+s^Y)2T}]$$

so the gain process is in reality working (although it is generated by an asset different than the bank's bond) and the replications strategy for the **DVA** (from the bank's perspective) is actually operating as expected. So, is the "funding benefit" argument correct? Should we then admit that the replication strategy suggested by Burgard and Kjaer [5] is right and that we were wrong above when we negated its effectiveness? As we hinted above, things are subtler then they may appear. Let us analyse which are the hidden assumptions under which the replication strategy is working.

Firstly, we have stated that the profit earned after that the bank's bonds is bought back is equal to the profit that the bank would have earned if it were able to actually buy its own bonds, and this was easily proved in the example above. We have assumed a constant spread, though, for both the bank $B$ and the issuer $Y$: this is the reason we can be sure that the profit generated by the bond $P^Y$ is exactly equal to that generated by the bond $P^B$. We can relax the assumption of constant spread by introducing for both issuers a more realistic time dependent spread $s_t^I$, $I \in \{B, Y\}$. But in this case we must make sure that, if spreads are a deterministic functions of time, they are the same function; alternatively, if funding spreads are stochastic processes, we have to impose they follow the same dynamics **and** they are perfectly correlated so that, starting from the same value, they evolve in the future precisely in the same way.

Secondly, we have to explicitly consider now the possibility of default for the bank and the issuer $Y$: actually the spread simply indicates that this probability is not zero. In an environment with no recovery upon default and no liquidity premium or intermediation costs, it is well known that $s^I = \lambda^I$, where $\lambda^I$ is as above the instantaneous default intensity for issuer $I$. Assume now that the condition of the identity of time functions for deterministic spreads, or of the perfect correlation for stochastic spreads, is fulfilled. Then equality $\alpha_t^B dP_t^B = \alpha_t^B dP_t^Y$ is guaranteed only if neither the bank $B$'s nor issuer $Y$'s default occurs in the interval of time $T$. Either default, though, affects differently the effectiveness of the replication strategy.

In fact, if bank goes bankrupt before the issuer $Y$, then the replication strategy would still work, although it will be very likely stopped, as the rest of the bank's activity, and

[11] Since the option is not fully replicated, has assumed above, the replication of the **DVA** is generating a gain different from zero. We have made this choice to highlight the contribution to the replica of the **DVA** of the long position in the asset $A_1$, that mimics a theoretic long bank's bonds positions.

the default procedure would start to pay creditors, if possible. So in this case, up to the bank's default time, the replication strategy works and afterwards it does not need to work anymore so that issuer $Y$ may go defaulted at any time without material consequences (for the limited scope of the replication, obviously).

If the issuer $Y$'s default occurs first, then the strategy is completely spoilt and the replication is not attained. Then, when default is considered, another condition we must add to the ones above, is that issuer $Y$'s default has to occur after the bank $B$'s default. We can slightly relax this assumptions and accept that they may happen together: also in this case the replication is attained up to the last instant needed by the bank and hence it does not produce negative consequences to the strategy.

Equipped with these results, we can recapitulate the conditions under which the "funding benefit" argument is valid and the the **DVA** (from the bank's perspective) is effectively replicated:

1. The spreads over the risk-free rate (the funding rate) of the asset and of the bank's bond must start at the same value at the inception of the replication strategy and they must be the same deterministic function of time or they must be commanded by two identical, perfectly correlated stochastic processes;

2. The times of the default of bank $B$ and issuer $Y$ must be perfectly correlated so that when one of the two goes defaulted, so does contemporaneously also the other.

These conditions are trivially fulfilled when the issuer $Y$ clashes with the bank $B$, but we know that in this case it is impossible for the bank to go long its own bonds. In other cases conditions can be only imperfectly, or not at all, fulfilled and the replication strategy will not be effective.

Besides, it should be stressed that the analysis we have presented refers to a very simplified situations, where the "bond" asset $A_1$ can be clearly isolated from the rest of the assets and its variations can be compared with those of the bank's **DVA**. In reality the assets' composition is quite complex and complicated, so that it would be an extremely hard task identifying which bond has to be considered to measure the funding benefit for a given derivative contract.

# 6 DVA Replication and Bank's Franchise

In bank management books, "franchise" is defined as the value that the bank is able to create from its branch network, its systems and people and from its customer base and brand. According to the widely supported "special information hypothesis" proposed by Bernanke [1], banks play a unique role in financial markets because they have private information about costumers unavailable to other, non-bank lenders (see also Black [2]). A different, not necessarily alternative, view is that banks' franchise value originates in their provision of liquidity and payments services to their customers. That is, banks are special institutions not because of their privileged information with respect to other lenders but because they can grant funds more easily than other economic operators. The hypothesis is presented and tested in Weisbrod *et al.* [12].

However originated, banks create a franchise if they are able to buy assets yielding more than required by the risks they embed. This can be done, for instance, by buying assets dealing mis-priced in the market: the franchise value of the bank is increased by the skills[12] of the traders and assets managers in this case. Although this is possible, it is very difficult it happens if financial markets are efficient as they can be assumed to be for liquid assets. For example, let us go back to the case we have analysed in the previous section: if we assume that the spread over the risk-free rate yielded by the bond $P^Y$ is due only to the default risk and that the recovery is zero, so that $s^Y = \lambda^Y$ (the notation is the same as above), then if the market prices correctly the risks, the expected return over a small period $dt$ is:

$$\mathbf{E}[dP_t^Y] = \mathbf{E}[(r_t + s_t^Y)P_t^Y dt - dJ^Y P_t^Y] = r_t P_t^Y dt$$

So in this case the bank franchise is not really increasing, even if the spread over the risk-free rate is positive. On the contrary, the bank is actually loosing money in expectation, since the funding spread on bank's liabilities has to be paid anyway, so that they accrue instantaneously interests at a rate $r_t + s_t^B$ with certainty[13] and the asset $A_1 = P^Y$ yields just an expected risk-free rate $r_t$.

Another way to create the franchise is to charge a margin over the fair rate that remunerates risks and costs, and that provides for a profit, when the bank lends money to clients that have a weaker bargaining power: those are especially retail ones that do not have an easy direct access to the capital markets. Going back to the case above, let us assume that the bond $P^Y$ is issued by a very particular obligor who is default risk-free and cannot have access to the capital market but can borrow money only from bank $B$. In this case the bank may apply the spread $m$ over the risk-free rate which is simply a margin and not a remuneration for the default risk. This means that the expected return on the asset $A_1 = P^Y$ is $r + m$. So, if $m = s^B + m' > s^B$ then the bank is increasing its franchise since it is able to generate profits in the future on a sound basis covering its funding costs. This is true also if the bank is lending money to defaultable obligors if it is able to charge a spread $s^Y = \lambda^Y + s^B + m'$ that remunerates the costs and the risks (*i.e.*: the bank's funding spread $s^B$ and the default risk $\lambda^Y$) and includes a positive margin $m$.

When we considered above the two conditions under which the **DVA** (from bank's perspective) can be effectively replicated, we mentioned the fact that the default times of the bank and of the obligor $Y$ must be perfectly correlated. This condition is tantamount, from the bank's perspective, to assume the possibility to buy an asset that is default-risk-free and yet that yields more than the risk-free rate. In fact when bank $B$ does not go defaulted, so does not also the obligor, hence when pricing the asset issued by $Y$ and evaluating it against the costs and the risks borne by the bank, the obligor's default does not need to be considered. Under these conditions, if the spread $s^Y > s^B$ then the bank is actually creating a franchise, notwithstanding the asset is defaultable. If $s^Y = s^B$ then the bank is just covering the funding costs without any profit margin. This is a

[12] We include "luck" in the skills. Actually "luck" is the greatest skill a trader could ask to be gifted with, in our opinion.

[13] The bank should consider itself a risk-free operator when evaluating its investments and when drawing its balance sheet. See Castagna [6] for a more detailed discussion on this.

very hypothetic and hardly realistic situation, but if this happens then the bank power to apply this rate over the risk-free rate (that could produce also a franchise) is "used" to replicate the **DVA**, thus confirming what we had said above, when we affirmed that replication of the **DVA** would end up, considering the entire balance sheet, with a cost that has to be covered by a margin above the risk-free rate on some other contracts.

Since the perfect correlation between the funding spreads of the obligor $Y$ and the bank $B$, jointly with the perfect correlation between the times of default for both, are very likely not matched in reality, the spread $s^Y$ is the remuneration for the default risk of the obligor $Y$, so that it cannot be "used" for the replication of the **DVA**. On the other hand, if the bank is able to apply a margin over the rate needed to remunerate the default risk, so as to compensate the funding costs $s^B$, this margin can be effective in replicating the **DVA**, although the bank should be able to update it frequently so as to track the variations of its own funding spread. In other words, the assets cannot be fixed rate bonds, and the spreads have to be reviewed not only to reflect the obligor's default risk but also the bank's default risk. Also in this case the bank is using its ability to finance some investments to cover losses represented by the **DVA**. So the **DVA** is formally hedged, but the cost has been indirectly charged to other business' areas and eventually, considering the total level of funding available, the bank will bear always the same total funding cost.

In the end, if when closing a derivative contract the bank receives some cash, this can be used to buy back a quantity of bank's own bonds. In this case the balance sheet shrinks, because an asset (the cash received) is used to abate liabilities (bank's bonds): given the reduced amount of liabilities, there is a smaller cost to pay, and this will be equal to the **DVA** of the derivative contract (under the stated conditions). If the bank was able, with bonds issued before the closing of the derivative contract,to buy assets yielding more than the risk-free rate on a risk-adjusted basis, and this extra-yield was enough to cover the funding costs over the risk-free rate of the bank's bonds, then it will be enough to cover also the cost of the **DVA**. There is nothing special or a funding benefit here, simply reduced liabilities (balancing reduced assets) produce smaller funding and **DVA** costs.

The problem to consider in a naive way the **DVA** as a funding benefit arise more clearly when derivative contracts do not produce any positive cash-flow, as for example when the bank deals a forward or a swap contract. In these kind of contracts, starting with zero value to both parties, the **DVA** can be either paid immediately to the counterparty, and there is no way to treat it differently than a cost; or it can be embedded in the value of the contract by modifying the fair forward or swap price so that it is worse than the risk-free equivalent for the bank.[14]. In this second case the bank can include in the balance sheet the value of the contract without separating the **DVA** component and treating it as a cost but, on the contrary, still relying on the funding benefit argument, considering it replicable by a buy-back of its own bonds as explained above.

It is easy to check that in this case it is impossible to buy back bonds and then consider the **DVA** as a funding benefit, and we will show why: assume the bank sold forward at

[14] Castagna [6] shows a simplified example of how a forward fair price would be modified when **DVA** is included in the value of the contract.

| Contract | | Time 0 | $T$ |
|---|---|---|---|
| Forward | Cash | - | $+\widehat{F}$ |
| | Asset | - | $-1$ |
| Repo | Cash | $S$ | $-Se^{rT} = -F$ |
| | Asset | $-1$ | 1 |
| Spot | Cash | $-S$ | - |
| | Asset | 1 | - |
| Net | Cash | 0 | $\widehat{F} - F < 0$ |
| | Asset | 0 | 0 |

Table 1: Hedging of a short forward contract by a sell-and-by-bak repo contract: position in the asset and in cash of the bank at the start of the contract and at the expiry.

future time $T$ to a risk-free counterparty an asset $S$ and that the fair risk-free forward price is $F = Se^{rT}$. Only the default risk of the bank has to be included in the valuation, so that the new forward price making nil the value of the contract at inception including also the **DVA** will be some $\widehat{F} < F$. In table 1 we show the how to hedge the forward contract and related positions in the underlying asset and in cash at the start and at the expiry. The replication is attained with a strategy whereby the bank sells-and-buys-back the asset $S$ by a repo transaction expiring in $T$ as well (we work under the assumption made above that the repo rate clashes with the risk-free rate). To deliver the underlying to the repo counterparty, the bank buys the asset on the spot market using the cash it receives from the repo sell-leg. Then at the contract expiry the bank has to buy the the underlying back from the repo counterparty: it will pay $Se^{rT} = F$ against receiving one unit of underlying, which will be delivered to the forward counterparty receiving an amount if cash equal to the forward price $\widehat{F}$. The net result is that the bank looses an amount of money $\widehat{F} - F$ that just equals the **DVA**.[15]

If the bank strictly followed the dynamic strategy indicated in Burgard and Kjaer [5], it should also buy back some quantity of its own bonds, but since in a forward contract no cash is received at the inception by the bank, the purchase can be financed only by resorting to a loan in the market, hence generating the ineffectual situation to replace bank's debt with equivalent bank's debt producing no result at all.

It would be better, to keep things clear, to consider in any case the **DVA** of the derivative contracts and of the issued bonds (for which **DVA** is exactly the funding cost) as costs reducing the equity. If positive cash-flows are received for whatever reason by the bank, then it can shrink the balance sheet by buying back outstanding debt, thus paying less funding costs on remaining liabilities. In this case, if assets are generating an extra-yield covering the funding costs of bonds, after they are bought back the same extra-yield can be used to cover the **DVA** of the derivative contracts. In no positive cash-flow occurs

[15]Things are in reality more complex since the repo rate is different from the risk-free rate to account for the default risk of the bank (in the sell-and-buy-back we are considering) and the the possibility that the underlying asset price is not fully covering he amount of the cash lent by the counterparty. We disregard all this here since it is beyond the scope of the analysis.

from the derivative transactions, the balance sheet cannot be shrunk and both the **DVA** and the funding costs of the outstanding debt are cost to be paid in the future.

In conclusion, if we disregard the need hardly met to update continuously the spread for the assets and the fact that not always positive cash-flows are received at inception, a bank that does not recognize the **DVA** as a cost when booking its derivative contracts, based on the false idea that it can be replicated, is implicitly using the margins that it is able (if ever it is so) to charge on other products (typically of the banking book) to cover costs generated by derivatives desks (trading book). The simple buy-back of its own bonds by the bank is not enough to justify the "funding benefit" argument if it is not sure that amongst the assets other contracts are covering the funding costs.

So, if the financial institutions is only an investment bank trading in the financial markets, the derivatives desk would rely on the profits of other desks to cover the **DVA**, so that the bank is destroying the franchise (in case it is created). If the institution operates also as a retail bank, the derivatives desk would rely on the ability of the desks dealing contracts of the banking book to include funding costs as margins above the risk-free rate plus credit spreads in the pricing. Besides, if bank's spread is volatile, these margins should be reviewed frequently to align them to the current funding spread paid by the bank. This would hardly happen in reality so it is more likely that also in this case the bank would end up by destroying value, or by just covering the total funding costs. If the bank allows traders to implement replication strategies for the **DVA** and it does not recognize immediately this quantity as a cost, there are two immediate negative consequences.

Firstly, when traders implement replication strategies, the bank is using margins generated in profitable businesses to cover losses generated by the losing money derivatives business. In some cases this loss can be compensated by shrinking the balance sheet with cash-flows received. So at best the bank is not increasing its franchise, or at worse it is actually destroying it. This could be a very long and non manifest process, especially when long dated contracts are involved (*e.g.*: a swap book), but the bleeding will be inexorable.

Secondly, traders (and possibly sales people) think they can hedge the **DVA** and they do not consider it as a cost they paid, so they will not try and transfer it to some other clients when dealing with them. If the bank is not able to avoid paying on some trades the **DVA**, and besides to charge on these trades the **DVA** of other deals for which it has to pay it, then the derivatives business is a loosing one, so it is better to close it. It is the same as if the bank keeps on lending money when it is not able to transfer to clients its funding spreads: sooner or later, the bank ends up loosing money.

# 7 Final Remarks

We have shown that the **DVA** cannot be replicated by a dynamic strategy, or it can be only under very unrealistic assumptions justifying the "funding benefit" argument. This does not mean that we do not have to consider bilateral counterparty risk when pricing a derivative contract. Neglecting transaction costs, which in practice also add to the final price of the contract, during the bargaining process each party tries to include into the price the relevant risks it bears, so each one considers also the adjustment due to the counterparty credit risk, or the **CVA**. This quantity can be replicated without any theoretical and practical hindering, since it is possible to trade short bonds issued by the other party. From the other's party perspective, the **CVA** is the **DVA** and since it cannot be replicated with a suitable dynamic strategy, it has to be considered a cost when included in the dealing price.

The price of the contract is an *obejctive* quantity, given by the level at which the two parties agree to trade the contract; the value of the contract is a *subjective* measure, and it is given by how much a contract is worth to one party. When the bank wants to compute which is the value of the contract, it has to consider itself as a risk-free operator. The same is true for the counterparty. This does not mean that funding costs are not considered in the evaluation process, simply one's own event of default has to be excluded: this leads to cancelling the **DVA** component in the bilateral counterparty credit risk component from the traded price to determine the value to the party.

Let $\mathbf{CVA}^B$ be the credit value adjustment due to the default risk of the bank $B$, which equals $\mathbf{DVA}^B$ or the debit value adjustment seen from the bank's perspective; analogously let $\mathbf{CVA}^C$ be the credit value adjustment due to the default risk of the counterparty $C$, equal to $\mathbf{DVA}^C$ or the debit value adjustment seen from the counterparty's perspective. Since each party considers itself default-risk-free in the evaluation process, the value of the contract to $C$ is:

$$V - \mathbf{CVA}^B$$

On the other hand, the value of the contract to $B$ is:

$$-(V + \mathbf{CVA}^C)$$

Let us consider the price of the contract in absolute value, and take the absolute values of the value of the contract to each party. Both parties may acknowledge that the other party is bearing the counterparty risk related to its default, even if when determining the value of the contract they exclude their own default risk. In practice each party knows that it may be forced to yield something with respect to the value of the contract.

Alternatively said, each party will trade at a price that will include a fraction (possibly the entire) **CVA** charged by the counterparty, which is to say: a price including a fraction of a quantity that cannot be replicated given by its **DVA**. Let $0 \leq \gamma^B \leq 1$ be the fraction of the $\mathbf{CVA}^B \equiv \mathbf{DVA}^B$ acknowledged to the bank by the counterparty, and $\gamma^C$ the fraction of $\mathbf{CVA}^C \equiv \mathbf{DVA}^C$ acknowledged to the counterparty by the bank. The trading price $p$ will be then:

$$V - \mathbf{CVA}^B \leq p \leq V + \mathbf{CVA}^C$$

where

$$p = V - \gamma^B \mathbf{CVA}^B + \gamma^C \mathbf{DVA}^C \equiv V + \gamma^C \mathbf{CVA}^C - \gamma^B \mathbf{DVA}^B$$

When both parties yield 100% of their respective **DVA** to the other party (*i.e.*: both parties can fully charge the **CVA** and fully accept the other party's **CVA**), then we have that the traded price $p$ is equal to the price including full bilateral counterparty risk and the pricing formula would be the same as in Brigo and Capponi [3]. The pricing formula would be also the same as the one derived by Burgard and Kjaer [5] and shown above, if one does not consider the funding costs related to the implementation of the replication stretegy (third line in equation (10)). This price can be defined as the fair dealing price.

Funding costs are specific to each party and are originated by the same default risk that originates the **DVA**. If we consider also funding costs, then the value to each counterparty would be even further away from the fair price at which they can deal in the market. Denoting with $\mathbf{FC}^B$ the present value of funding costs paid by the bank $B$ in replicating the contract, and analogously with $\mathbf{FC}^C$ the present value of funding costs paid by counterparty $C$, we can generalize the bounds within which the price of the contract deals:

$$V - \mathbf{CVA}^B - \mathbf{FC}^C \leq p \leq V + \mathbf{CVA}^C + \mathbf{FC}^B$$

and the dealing price of the contract $p$ is now:

$$\begin{aligned} p &= V - \gamma^B \mathbf{CVA}^B + \gamma^C \mathbf{DVA}^C - \zeta^C \mathbf{FC} + \zeta^B \mathbf{FC}^B \\ &\equiv V + \gamma^C \mathbf{CVA}^C - \gamma^B \mathbf{DVA}^B - \zeta^C \mathbf{FC} + \zeta^B \mathbf{FC}^B \end{aligned}$$

$\zeta^C$ and $\zeta^B$ are the fraction of the present value of the funding costs born by counteparty $C$ and by bank $B$ acknwoledged by, respectively, bank $B$ and counterparty $C$.

To conclude, when default risk is considered, even in absence of transaction costs, a unique value fair to both counterparties is not possible to be determined. The fair dealing price, if attained during the bargaining process, would be in any case a level different from both values to parties. All the adjustments referring to the counterparty risk can be replicated by the parties, but this does not mean that any adjustment can be replicated by any party: in fact the **DVA** of each party can be replicated only by the other one, for which it is the **CVA**.

The ultimate conclusion is that derivatives trading, amongst professionals with even bargaining power and without credit mitigation mechanisms, is a loosing business for banks unless they trade also with weaker parties whom they can transfer **DVA** related costs (and funding costs) to.

## References


[1] B. S. Bernanke. Nonmonetary effects of the financial crisis in the propogation of the great depression. *American Economic Review*, June:257 – 276, 1983.

[2] F. Black. Bank fund management in an efficient market. *Journal of Financial Economics*, 2:323 – 339, 1975.

[3] D. Brigo and A. Capponi. Bilateral counterparty risk valuation with stochastic dynamical models and applications to credit default swap. *Risk*, March, 2010.

[4] D. Brigo and M. Morini. Dangers of bilateral counterparty risk: the fundamental impact of closeout conventions. *Working Paper, avilable at http://arxiv.org*, 2010.

[5] C. Burgard and M. Kjaer. Pde representations of options with bilateral counterparty risk and funding costs. *Available at http://papers.ssrn.com*, 2010.

[6] A. Castagna. Funding, liquidity, credit and counterparty risk: Links and implications. *Iason research paper. Available at http://iasonltd.com/resources.php*, 2011.

[7] D. Duffie. *Dynamic Asset Pricing Theory*. Princeton University Press, third edition, 2001.

[8] R.C. Merton. *Continuous-Time Finance*. Basil Blackwell Inc., 1990.

[9] M. Morini and A. Prampolini. Risky funding with counterparty and liquidity charges. *Risk*, March, 2011.

[10] A. Pallavicini, D. Perini, and D. Brigo. Funding valuation adjustment: a consistent framework including cva, dva, collateral, netting rules and re-hypothecation. *Available at http://defaultrisk.com*, December, 2011.

[11] E. Severino. *La Struttura Originaria*. Adelphi Edizioni, 1981.

[12] S.R. Wiesbrod, H. Lee, and L. Rojas-Suarez. Bank risk and the declining franchise value of the banking systems in the united states and japan. *IMF Working Paper*, 92/45, 1992.