Pricing Swaps Including Funding Costs

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1 Introduction

In Castagna [3] we have tried to correctly define the Debit Value Adjustment (**DVA**) of a derivative contract, coming up with a definition that declares the **DVA** the worsening of contract conditions for a counterparty because it has to compensate the other party for the possibility of its own default. The **DVA** is very strictly linked to funding costs (**FC**) when the contract is a loan, a bond or more generally some kind of borrowing. The link is much less tight, and in fact it could even be non existent, for some derivatives contracts such as swaps. The funding costs for a derivative contract is actually the **DVA** (plus liquidity premium and intermediation cost, if priced in market quotes) that a counterparty has to pay on the loan contracts it has to close to fund, if needed, negative cumulated cash-flows until maturity.[1]

In this paper we study how to include funding costs into the pricing of interest rate swaps and we show how they affect the value of the swap via a Funding Value Adjustment (**FVA**), in analogy with the Credit Value Adjustment (**CVA**) and the **DVA**. In what follows we consider the pricing of swap contracts with no collateral agreement or any other form of credit risk mitigations.

2 The Basic Set-Up

Assume that, at time t, we want to price a very general (non-standard) swap, such as an amortizing or a zero-coupon swap, with possibly different amounts for the fixed and the floating rate, and with a also possibly time-varying fixed rate.

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[1] See Castagna [3] for more details and the assumption underpinning these definition. We will work under the same assumptions also in what follows.

Let us introduce a meta-swap, which is a swap with unit notional and time varying fixed rate that is equivalent to the contract fixed rate times the notional amount for each date N^K_{i-1} (*i.e.*: that one at the start of the calculation period). The start date of the swap is T_a and the end date is T_b.

Let us assume that the swap's floating leg pays at times $T_{a+1}, \dots, T_b$, where T_{a+1} is the first fixing time (we assume that dates are equally spaced given the floating leg payment frequency); $F_i(t)$ are the forward rates, as of time t, paid at time T_i and fixed in T_{i-1}, for $a+1 \leq i \leq b$; the swap's fixed leg pays at times $T_{c_1}, \dots, T_{c_J}$, where $c_1 \geq a$ and $c_J = b$. The fixed leg times are assumed to be included in the set of floating leg times and this is usually the case for standard swaps quoted in the OTC market, for which the floating flows are paid semi-annually or quarterly, whereas the fixed flows are paid annually.

The fixed rate payment at each payment date T_{c_j} is:

$$R_j = \beta_j K \tag{1}$$

where

$$\beta_j = N^K_{j-1}\delta^K_j \tag{2}$$

and δ^K_j denotes the year fraction between to payment dates for the fixed leg.

The floating leg will exchange the future Libor fixing times α_i, which is the year fraction times the notional N^L_{i-1} at the beginning of the calculation period:

$$\alpha_i = N^L_{i-1}\delta^L_i \tag{3}$$

Note that despite the fact that the meta-swap has unit notional, both the total fixed rate and the year fraction are containing the notional of the swap.

Define

$$\bar{C}_{a,b}(t) = \sum_{j=1}^{J} \beta_j P(t, T_{c_j}) \tag{4}$$

as the annuity, or the DV01 in the market lore, of the meta-swap. We assume $\bar{C}_{a,b}(t) > 0$. The discount factors (or discount bonds) $P(t,T)$ are taken from a risk-free curve; in the current market environment, the best approximation to the risk-free rate is given by the overnight rates. An entire curve based on these rates can be bootstrapped from OIS swaps. Define also:

$$w_i(t) = \frac{\alpha_i P(t, T_i)}{\bar{C}_{a,b}(t)} \tag{5}$$

We then have:

$$S_{a,b}(t) = \sum_{i=a+1}^{b} w_i(t) F_i(t) \tag{6}$$

which is the swap rate that makes nil the value of the the meta-swap in t, $\mathbf{Swp}_{a,b}(t) = 0$ ($\mathbf{Swp}_{a,b}(t)$ is the value at time t of a swap starting in T_a and terminating in T_b). In a standard swap the fair rate is the average of the forward Libor rates F_j weighted by a function of the discount factors. In the case of the meta-swap the average of the forward Libor rates is weighted by a function of the notionals and discount factors. It can be easily checked that this is the rate making the present value of the floating equal to that of the fixed leg. It should be stressed that the risk-free rates used to derive the discount factors are not the same used to determine the Libor forward rates F_j; for more details on the new pricing formulae to adopt after the financial crisis of the 2007, see Bianchetti [1] and Mercurio [6].

Some points should be stressed. First, the pricing is correct if the both counterparties involved are risk-free; secondly, since at least one of the two counterparties is usually a bank, the fact that the Libor rates are above the risk-free rates is in conflict with the first point, Libor being rates applied to unsecured lending to an ideal bank with a good credit rating, but not risk-free in any case; thirdly, as a consequence of the second point, a full-risk pricing should include also the credit adjustments (**CVA** and **DVA**) as a compensation of the default risk referring to either parties.

To isolate the funding component of the value of a swap, we operate at this point an abstraction and we do not consider the adjustments due to counterparty credit risk. The methodologies to include them into the pricing have been examined in some works, such as Damiano and Capponi [2]. To analyse the problem linked to the cost of funding, we first introduce a hedging strategy for the swap and then we analyse the cash-flows implied by it.

3 Hedging Swap's Exposures and Cash-Flows

Assume a bank takes a position in a swap starting in T_a and ending in T_b, that can be described by the general formulae we have seen above: the fair swap rate is $\bar{S}_{a,b} = S_{a,b}(t)$. The swap can be either payer (receiver) fixed rate, in which case the fixed leg has a negative (positive) sign. The bank wants to hedge the exposures with respect to the interest rates, but also it wants to come up with a well-defined, possibly deterministic, schedule of cash-flows so as to plan their funding and/or investment. To lock in future cash flows, we suggest the following strategy:

- Take all the dates $T_{c_1}, \ldots, T_{c_J}$, when fixed-leg payments occur;
- Close (forward) starting swaps $\mathbf{Swp}(T_{c_{i-1}}, T_{c_i})$, for $i = 1, \ldots, J$ with fixed-rate payments opposite to those of the swap the bank wants to hedge. The fair rate for each swap is $\bar{S}_{c_{i-1},c_i} = S_{c_{i-1},c_i}(t)$.

Define now $\mathbf{CF}(T_k)$ as the amount of cash to receive or to pay at time T_k, generated by the hedged portfolio above. The floating leg of each hedging swap is balancing the floating leg of the meta-swap for the corresponding period, so that at each time T_i, with $a+1 \leq i \leq b$ we have that $\mathbf{CF}(T_i) = 0$. On the dates T_{c_j}, for $1 \leq j \leq J$, when the fixed legs of the total portfolio (comprising the meta-swap and hedging swaps) are paid, the net cash-flows are:

$$\mathbf{CF}(T_{c_j}) = (\mathbf{1}_{\{R\}} - \mathbf{1}_{\{P\}})\bar{S}_{a,b} - (\mathbf{1}_{\{R\}} - \mathbf{1}_{\{P\}})\bar{S}_{c_{i-1},c_i}$$

where $\mathbf{1}_{\{R\}}$ (respectively, $\mathbf{1}_{\{P\}}$) is the indicator function equal to 1 if the swap is receiver (respectively, payer).

Define also $\mathbf{CCF}(T_a, T_{c_j})$ as the compunded cumulated cash-flows from the start time T_a up to time T_{c_j}:

$$\mathbf{CCF}(a, c_j) = \sum_{k=1}^{j} \mathbf{CF}(T_{c_k}) \frac{P(t, T_{c_{k-1}})}{P(t, T_{c_k})} \tag{7}$$

Cash-flows are assumed to be reinvested at the risk free rate: this is possible if the cumulated cash flows start at zero, increase and do not become negative. We indicate by $\mathbf{CF}(c_k)^{\pm}$ a positive/negative cash flow, whereas we indicate with $\overline{\mathbf{CCF}}(a, b)$ the maximum amount of cumulated cash-flows between the start date T_a and the end date T_b:

$$\overline{\mathbf{CCF}}(T_a, T_b) = \max[\mathbf{CCF}(T_a, T_{c_1}), \mathbf{CCF}(T_a, T_{c_2}), ..., \mathbf{CCF}(T_a, T_{c_J})] \tag{8}$$

Analogously we denote with $\underline{\mathbf{CCF}}(T_a, T_b)$ the minimum amount of cumulated cash-flows:

$$\underline{\mathbf{CCF}}(T_a, T_b) = \min[\mathbf{CCF}(T_a, T_{c_1}), \mathbf{CCF}(T_a, T_{c_2}), ..., \mathbf{CCF}(T_a, T_{c_J})] \tag{9}$$

For standard market swaps, we generally have two possible patterns of the cumulated cash-flows, depending on the side of the swap (fixed rate payer/receiver) and on the shape of the term structure of interest rates: the first pattern is always negative, while the second is always positive. This means also that $\overline{\mathbf{CCF}}(T_a, T_b)$ is zero and $\underline{\mathbf{CCF}}(T_a, T_b)$ is a negative number in the first case; in the second case $\underline{\mathbf{CCF}}(a, b)$ is zero and $\overline{\mathbf{CCF}}(a, b)$ is a positive number. As far as funding costs have to be included into the pricing, we have to focus only on the first case, whereas the second case poses no problems. In fact, in the second case, the cash-flows generated internally within the deal, including their reinvestment in a risk-free asset, imply no need to resort to additional funding. This is not true in the first case.

Negative cash-flows need to be funded and the related costs should be included into the pricing. As mentioned above, somewhat inconsistently, we do not consider the effect of the defaults of either parties on funding costs.

Now, given the market term structure of forward Libor rates, a swap usually implies for a counterparty a string of negative cash flows compensated by a subsequent string of positive cash flows. The present (or, equivalently, the future at expiry) value of negative cash-flows is equal to the present, or future, value of positive cash-flows, provided there is no default of either counterparties, and that each counterparty is able to lend and to borrow money at the risk-free rate.

If we assume that it is possible, for the counterparties, to lend money at the risk-free rate, but that they have to pay a funding spread over the risk-free rate to borrow money, then the problem of how to correctly consider this cost arises. We suggest two strategies to fund negative cash-flows, the second one in two variants. We examine them separately from the perspective of one of the two parties, let us say the bank, whereas the other party is assumed to be a client that is not able to transfer his/her funding costs into the pricing.

4 Funding Spread Modelling

To keep things simple, we assume that the funding spread is due only to credit factors and there are no liquidity premiums. More specifically, the bank has to pay a spread that originates from its default probability and the loss given default. If we assume that after default a fraction $\mathcal{R}$ of the market value of the contract is immediately paid to the counterparty (Recovery of Market Value (RMV) assumption) then we have a very convenient definition of the instantaneous spread (see Duffie and Singleton [5]) as $\varsigma_t = (1 - \mathcal{R})\lambda_t$, where λ is the default intensity, *i.e.*: the jump intensity of a Poisson process, the default being the first jump. We choose a doubly stochastic intensity model so that the survival probability between time 0 and time T is given by:

$$Q(0, T) = e^{-\int_0^T \lambda_s ds}$$

where default intensity λ_t is a stochastic process that is assumed to be commanded by the CIR-type dynamics:

$$d\lambda_t = \kappa_\lambda(\theta_\lambda - \lambda_t)dt + \sigma_\lambda\sqrt{\lambda_t}dZ_t \tag{10}$$

In this setting, $Q(0, T)$ has a closed form solution (see Cox, Ingersoll and Ross [4]):

$$Q(0, T) = E^Q\left[\exp\left(-\int_0^T \lambda_s ds\right)\right] = A(0, T)e^{-B(0,T)\lambda_0} \tag{11}$$

$$A(0,T) = \left(\frac{2\xi e^{\frac{(\xi+\kappa_\lambda+\psi_\lambda)T}{2}}}{(\xi+\kappa_\lambda+\psi_\lambda)(e^{\xi T}-1)+2\xi} \right)^{\frac{2\kappa_\lambda\theta_\lambda}{\sigma_\lambda^2}}$$

$$B(0,T) = \frac{2(e^{\xi T}-1)}{(\xi+\kappa_\lambda+\psi_\lambda)(e^{\xi T}-1)+2\xi}$$

$$\xi = \sqrt{(\kappa_\lambda+\psi)^2+2\sigma_\lambda^2}$$

We set the premium for market risk $\psi_\lambda = 0$ in what follows.

The formula to compute the spread discount factors can be easily shown to be the same as for the survival probability with a slight change of the parameters:

$$P^s(0,T;\lambda_0,\kappa_\lambda,\theta_\lambda,\sigma_\lambda,\mathcal{R}) = Q(0,(1-\mathcal{R})T;\lambda_0,\frac{\kappa_\lambda}{1-\mathcal{R}},\theta_\lambda,\frac{\sigma_\lambda}{\sqrt{1-\mathcal{R}}}) \tag{12}$$

Let $P(0,T)$ be the price in 0 of a default risk-free zero coupon bond (bootstrapped from the OIS swap curve, as an example) maturing in T; the price of a correspondent zero coupon bond issued by the bank is $P^D(0,T) = P(0,T)P^s(0,T)$ (where we have omitted some parameters of the function $P^s(0,T)$ to lighten the notation), assuming a default intensity given by the dynamics in (10) and a recovery rate $\mathcal{R}$. This is also the discount factors used to compute the present value of money borrowed by the bank, and it should be considered as effective discount factor embedding also funding costs.[2]

4.1 Strategy 1: Funding All Cash-Flows at Inception

The first strategy is based on the idea to fund all negative cash-flows right from the inception of the swap. To this end, we compute the minimum cumulated amount $\underline{\mathbf{CCF}}(a,b)$ over the entire duration of the swap $[T_a,T_b]$. Assuming that $\underline{\mathbf{CCF}}(a,b) < 0$, this is the amount that needs to be entirely funded at the inception. The idea is to borrow money and then use the cash-flows generated by the hedged swap portfolio to repay it, possibly also according to a predefined amortization schedule determined by the cash-flows' pattern. We need to consider some relevant practical matters too:

- The total sum that is entirely funded at the inception can be invested in a risk-free asset (a zero-coupon bond issued by a risk-free counterparty,[3] for example). The amounts needed when negative cash-flows occur can be obtained by selling back a fraction of the investment. The interests earned have to be included in the pricing.
- The funding for long maturities can be done with a loan that the bank trade with another counterparty; this usually implies a periodic payment of interests on the outstanding amount. Also these periodic paid interests need to be included in the evaluation process.

[2] See Castagna [3] for a discussion on this point.

[3] When considering defaultable issuer, their debt should be remunerated by a spread over the risk-free rate to compensate for the risk of default, so that ultimately the expected return is still the risk-free rate anyway.

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To formalize all this, consider that the amount borrowed by the bank at the inception t_0 is A. The bank pays annual interests on the outstanding of the borrowed amount on an annual basis, according to a fixed rate calculated at the start considering also the probability of default. We assume that the banks pays a fraction of the market value of the loan on the occurrence of its default.

Let $t = 0$ and A be the initial amount of a loan that expiries in T_b (equal to the expiry of the swap) and it has a capital and interest payment schedule in dates $[T_{d_1}, \ldots, T_{d_M}]$: we assume that this set contains also the set of payment dates for the fixed leg of the swaps. We define the capital payment of the loan A at time T_k as $K(T_k) = A(T_k) - A(T_{k-1})$, with $A(t) = A$, $A(T_b) = 0$ and $\sum_{k=1}^{M} K(T_k) = A$. It should be noted that the loan starts at the inception of the contract t, that could be also before the start of the swap T_a; besides interest payments can also occur before T_a. Let $\bar{i}$ be the fixed rate that the bank has to pay on this loan: it can be derived from the following relationship

$$A = \sum_{k=1}^{M} (K(T_k) + \bar{i} A(T_{k-1})\delta_k) P^D(0, T_k) \tag{13}$$

where $\delta_k = T_k - T_{k-1}$ is the accrual period. The discounting is operated by means of the discount factors $P^D(T_0, T_k)$ to account also for the losses the lender suffers on bank's default. From the bank's perspective the spread paid over the risk-free rate is a funding cost, whereas the same is the compensation for the the default risk borne from the lender's perspective. [4] The loan's fair fixed rate $\bar{i}$ is:

$$\bar{i} = \frac{A - \sum_{k=1}^{M} K(T_k) P^D(0, T_k)}{\sum_{k=1}^{M} A(T_{k-1}) \delta_k P^D(0, T_k)} \tag{14}$$

As mentioned above, once the amount of the loan A is received by the bank at time 0, it can be reinvested at the risk free rate and partially reduced to cover future outflows of cash when they occur. Let us define the available liquidity at time T_{d_k} via the recurrent equation:

$$\mathbf{AVL}(T_{d_k}) = \mathbf{AVL}(T_{d_{k-1}}) \frac{P(t, T_{d_{k-1}})}{P(t, T_{d_k})} + \mathbf{CF}(T_{d_k}) - K(T_{d_k}) - \bar{i} A(T_{d_{k-1}}) \tag{15}$$

with $\mathbf{AVL}(0) = A$. Equation (15) states that the liquidity, available for the bank at time T_{d_k}, is the liquidity available at the previous time $T_{d_{k-1}}$ invested at the forward risk-free rate over the period $[T_{d_{k-1}}, T_{d_k}]$, plus the cash-flow occurring at time T_{d_k}, deducted the sum of installment and the interest rate payments. Cash-flows can be either positive or negative. We impose that when a positive cash-flow occurs, $\mathbf{CF}(T_{d_k}) > 0$, it is used to abate the outstanding amount of the loan; on the other hand, when a negative cash-flow occurs, $\mathbf{CF}(T_{d_k}) < 0$, then there is no capital installment and $C(T_{d_k}) = 0$. Since it is possible to lock in the future cash-flows at contract's inception via the suggested hedging portfolio, the amortization plan for the loan, however irregular it may be, can be established at time $t = 0$. The amortization plan can be defined then as:

$$A(T_{d_k}) = A(T_{d_{k-1}}) - \mathbf{CF}^+(T_{d_k})$$

[4] See Castagna [3] for a more detailed discussion.

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The amount of the loan that the bank has to borrow will be a function of the term structure of Libor interest rates and of the bank funding spreads, the fixed-leg notional schedule of the swap and the fixed rate of the swap:

$$A = f(F_1(0), \ldots, F_b(0), s_1(t), \ldots, s_b(t), N_1^K, \ldots, N_J^K, \bar{S}_{a,b})$$

Where $s_k(t)$ is the funding spread for the period $[T_{k-1}, T_k]$. The amount A has to be determined so as to satisfy two constraints:

1. The available liquidity $\mathbf{AVL}(T_{d_k})$ at each time T_{d_k} has to be always positive, so that no other funding is required until the end of the swap.
2. At the maturity of the swap T_b the available liquidity should be entirely used to finance all negative cash-flows, so that $\mathbf{AVL}(T_b) = 0$, thus minimizing funding costs (no unnecessary funding at inception has been required by the bank).

The amount A can be determined very quickly numerically. Given a positive funding spread, the positive cash-flows originated by the hedged portfolio will not be sufficient to cover entirely the loan's amortization plan, so that on the last capital installment date an extra cash must be provided by the bank to pay back entirely its debt and this represent ultimately a cost and it has to be included into the pricing of the swap. Let **FC** be the present value of this cost, then it can be added into the fair swap rate as follow:

$$S_{a,b}(0) = \sum_{i=a+1}^{b} w_i(0) F_i(0) + (\mathbf{1}_{\{R\}} - \mathbf{1}_{\{P\}}) \frac{\mathbf{FC}}{\bar{C}_{a,b}(0)} \tag{16}$$

where the annuity $\bar{C}_{a,b}(0)$ and the weights $w_i(0)$ are defined as as in (4) and in (5). Equation (16) increases (decreases) the fair swap rate if the bank is a receiver (payer) fixed rate in the contract, thus compensating the extra costs due to funding costs.

Since the amount of the loan A is a function of the swap rate $S_{a,b}(0)$, which in turn is affected by the funding cost **FC** that depends of A, a numerical search is needed to determine the final fair swap rate $\bar{S}_{a,b}^{\mathbf{FC}}$, which makes both the available liquidity and the **FC** equal to zero. The convergence is typically achieved in a few steps.

The value of the payer swap, when the rate is $\bar{S}_{a,b}^{\mathbf{FC}}$, is:

$$\mathbf{Swp}^{\mathbf{FC}}(T_a, T_b) = \sum_{i=a+1}^{b} w_i(0) F_i(0) - \bar{S}_{a,b}^{\mathbf{FC}} \bar{C}_{a,b}(0) = \mathbf{FVA} \tag{17}$$

Since $\bar{S}_{a,b}^{\mathbf{FC}} < \bar{S}_{a,b}$, the swap has a positive value that equates the funding value adjustment **FVA**, which is the quantity that makes the swap value nil at inception when funding costs are included into the pricing.

5 Strategy 2: Funding Negative cash-flows when They Occur

The second strategy we propose is matching negative cash-flows when they occur by resorting to new debt, given that cumulated cash-flows are not positive and/or insufficient. The debt is carried on by rolling it over and paying a periodic interest rate plus a funding spread; besides it can be increased when new negative cash-flows occur and decreased when positive cash-flows are received. Interest rates and funding spreads paid are those prevailing in the market at the time of the roll-over, so that they are not fixed at the inception of the contract.

The advantage of this strategy over the first one shown above, is that the bank borrows money only when it needs, and it does not have to pay any interest and funding spread for the time before cumulated cash-flows are negative. On the other hand, the bank is exposed to liquidity shortage risks and to uncertain funding costS that cannot be locked in from the start of the contract. We will show better the latter statement in what follows.

Assume that the hedged swap portfolio generates at a given time T_k a negative cash flow $\mathbf{CF}^-(T_k)$, and that cumulated cash-flows are negative: the bank funds the outflow by borrowing money in the interbank market. We assume that the debt is rolled over in the future and that the bank pays the interest plus a funding spread over the period $[T_k, T_{k+1}]$; the borrowed amount varies depending on the cash-flow occurring at time T_{k+1}. Hence the debt evolves according to the following recurrent equation:

$$\mathbf{FDB}(T_{k+1}) = \mathbf{FDB}(T_k)\frac{P^D(t,T_k)}{P^D(t,T_{k+1})} - \mathbf{CF}(T_{k+1}) \tag{18}$$

It is worth noticing that we are using the defaultable discount factors to include the interest payments over the period $[T_k, T_{k+1}]$. This means that we are forecasting the future total interests paid by the bank as the forward rates implicit in the Libor rates and the funding spreads at time $t = 0$. If the credit spread of the bank is positive, the positive cash-flows generated by the hedged portfolio will not be enough to cover entirely the payback of the debt and the related funding costs. The terminal amount left is, as in the first strategy proposed above, a cost that the bank has to pay that is strictly related to its credit spread. Ultimately this is a funding cost to include into the pricing of the swap.

The Libor component of the total interest rate paid can be hedged by market instruments (*e.g.*: FRAs), so that the implicit forward rates can be locked in. There is another component, though, that has to be considered: the forward funding spread, implicit in the defaultable bonds' prices, cannot be locked in easily at the start of the swap contract: this would entail for the bank trading credit derivatives on its own debt, which is either impossible (as in the case of CDS) or difficult (as in the case of spread options). The unexpected funding cost, due to the volatility of the credit spread of the bank, has to be measured in any case and it should be included into the pricing too. We suggest two possible approaches to measure the unexpected future funding costs.

5.1 Measuring Unexpected Funding Costs with Spread Options

The first approach we introduce is the measurement of the unexpected funding costs via spread options. Assume the roll-over of the debt is operated at dates $[T_{d_1}, \ldots, T_{d_M}]$, a set that contains also the set of dates of payments of the fixed leg of the swaps. The forward rate, computed in t, paid on the outstanding debt at a given date T_{d_k} is:

$$F^D_{d_k}(t) = \left(\frac{P^D(t,T_{d_{k-1}})}{P^D(t,T_{d_k})} - 1\right)\frac{1}{\delta_{d_k}} = (P^D_t(T_{k-1,d_k}) - 1)\frac{1}{\delta_k}$$

where $P_t^D(T_{d_{k-1}}, T_{d_k})$ is the forward price of the defaultable bond calculated in t. The expected funding cost at at time T_{d_k} is:

$$\begin{aligned}\mathbf{EFC}(T_{d_k}) &= \mathbf{FDB}(T_{d_{k-1}})F_{d_k}^D(t)\delta_{d_k} \\ &= \mathbf{FDB}(T_{d_{k-1}})\frac{1}{P_t^D(T_{d_{k-1}}, T_{d_k})} \\ &= \mathbf{FDB}(T_{d_{k-1}})\frac{1}{P_t(T_{d_{k-1}}, T_{d_k})}\frac{1}{P_t^s(T_{d_{k-1}}, T_{d_k})}\end{aligned} \tag{19}$$

Let $s_{d_k}(t)$ be the forward funding spread, linked to the spread discount factor as follows:

$$1 + s_{d_k}(t)\delta_k = \frac{1}{P_t^s(T_{d_{k-1}}, T_{d_k})} \tag{20}$$

so that

$$\mathbf{EFC}(T_{d_k}) = \mathbf{FDB}(T_{d_{k-1}})\frac{1}{P_t(T_{d_{k-1}}, T_{d_k})}s_{d_k}(t)\delta_{d_k} \tag{21}$$

As mentioned above, this is only the expected (under the forward risk survival measure measure[5]) funding spread. The unexpected part has to be considered and it can be written as:

$$\mathbf{UFC}(T_{d_k}) = \mathbf{FDB}(T_{d_{k-1}})\frac{1}{P_t(T_{d_{k-1}}, T_{d_k})}\max[s_{d_k}(T_{d_k})\delta_{d_k} - s_{d_k}(t)\delta_{d_k}; 0] \tag{22}$$

Equation (23) expresses the unexpected funding cost as a call spread option, with the strike equal to the forward spread calculated at time t. Clearly we are interested at the cases when the spread is above the expected forward level: if it actually is lower, then the bank will pay less then expected, but we do not consider this potential benefit here. It is possible, with a little algebra, to rewrite the equation in terms of an option on a discount bond:

$$\mathbf{UFC}(T_{d_k}) = \mathbf{FDB}(T_{d_{k-1}})\frac{1}{P_t(T_{d_{k-1}}, T_{d_k})}(1 + s_{d_k}(t)\delta_{d_k})\mathbf{ZCP}(1/(1 + s_{d_k}(t)\delta_{d_k}), t, T_{d_{k-1}}, T_{d_k}) \tag{23}$$

where **ZCP** is the future value, computed in t, of a put option with expiry $T_{d_{k-1}}$, on a zero coupon bond maturing in T_{d_k}, struck at $1/(1 + s_{d_k}(t)\delta_{d_k})$. The option is computed under the assumption that the default intensity is a mean reverting square root process, as described above. The solution for the present value of a call option expiring in T, written on a bond expiring in S, is provided by Cox, Ingersoll and Ross [4] and it is:

$$\begin{aligned}\mathbf{Call}(X, t, T, S) =& P_t(t, S)\chi^2\left(2\lambda^*(\phi + \eta + B(T, S); \frac{4\kappa_\lambda\theta_\lambda}{\sigma_\lambda^2}, \frac{2\phi^2\lambda_t\exp[\gamma(T-t)]}{\phi + \eta + B(T, S)}\right) \\ & XP_t(t, T)\chi^2\left(2\lambda^*(\phi + \eta); \frac{4\kappa_\lambda\theta_\lambda}{\sigma_\lambda^2}, \frac{2\phi^2\lambda_t\exp[\gamma(T-t)]}{\phi + \eta}\right)\end{aligned}$$

[5] The forward risk survival measure uses the defaultable discount bond as numeraire. For more details see Schonbucher [7]. We would like to stress that we are measuring funding costs under a *going-concern principle*, so that the bank does not take into account its own default into the evaluation process.

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where

$$\gamma = \sqrt{\kappa_\lambda^2 + 2\sigma_\lambda^2}$$

$$\phi = \frac{2\gamma}{\sigma_\lambda^2(\exp[\gamma(t' - t)] - 1)}$$

$$\eta = \frac{\kappa_\lambda + \gamma}{\sigma_\lambda^2}$$

$$\lambda^* = \left[\ln \frac{A(T,S)}{X}\right] \Big/ B(T,S)$$

For a put option one can use the put call parity $\mathbf{Put}(X, t, T, S) = \mathbf{Call}(X, t, T, S) - P_t(t, S) + XP_t(t, T)$. If the recovery rate $\mathcal{R}$ is different from 0, then the parameters have to be adjusted as follows:

$$\kappa_\lambda \to \frac{\kappa_\lambda}{1-\mathcal{R}},\ \ \sigma_\lambda \to \frac{\sigma_\lambda}{1-\mathcal{R}},\ \ t \to t(1-\mathcal{R}),\ \ T \to T(1-\mathcal{R}),\ \ S \to S(1-\mathcal{R})$$

The future value of the put option on the spread zero coupon bond is:

$$\mathbf{ZCP}(1/(1+s_{d_k}(t)\delta_{d_k}), t, T_{d_{k-1}}, T_{d_k}) = \frac{1}{P_t^s(T_{d_{k-1}}, T_{d_k})}\mathbf{Put}(1/(1+s_{d_k}(t)\delta_{d_k}), t, T_{d_{k-1}}, T_{d_k})$$

which inserted in (23) yields:

$$\mathbf{UFC}(T_{d_k}) = \mathbf{FDB}(T_{d_{k-1}})\frac{1}{P_t(T_{d_{k-1}}, T_{d_k})}\frac{1}{P_t^s(T_{d_{k-1}}, T_{d_k})}\mathbf{Put}(1/(1+s_{d_k}(t)\delta_{d_k}), t, T_{d_{k-1}}, T_{d_k}) \tag{24}$$

The total funding cost is the present value of the amount of the debt left at the expiry of the swap, that has to be covered by the bank and that is thus a cost, plus the present value of the spread options needed to hedge the unexpected funding costs for each period:

$$\mathbf{FC} = P(t, T_b)\mathbf{FDB}(T_b) + \sum_{k=1}^{M} P(t, T_{d_{k-1}})\mathbf{UFC}(T_{d_k}) \tag{25}$$

This quantity is then used to set determine, via a numerical search as in equation (16), the fair swap rate: this is the rate making nil the present value of the funding cost $\mathbf{FC} = 0$.

5.2 Measuring Unexpected Funding Costs with a Confidence Level

The second approach to measure unexpected funding costs is justified by the difficulty for the bank to buy options on its own credit spread. For this reason we suggest to consider the unexpected cost as a loss that cannot be hedged and that has to be covered by economic capital, similarly to the VaR methodology.

The expected funding cost is still the same as in formula (19). The unexpected cost is computed by

$$\mathbf{UFC}(T_{d_k}) = \mathbf{FDB}(T_{d_{k-1}})\frac{1}{P_t(T_{d_{k-1}}, T_{d_k})}[s_{d_k}^*(T_{d_k})\delta_{d_k} - s_{d_k}(t)\delta_{d_k}] \tag{26}$$

or, equivalently,

$$\mathbf{UFC}(T_{d_k}) = \mathbf{FDB}(T_{d_{k-1}})\frac{1}{P_t(T_{d_{k-1}}, T_{d_k})}\left[\frac{P^{s^*}(t, T_{d_{k-1}})}{P^{s^*}(t, T_{d_k})} - \frac{P^s(t, T_{d_{k-1}})}{P^s(t, T_{d_k})}\right] \tag{27}$$

The price of the spread discount bond $P^{s^*}(t, T_{d_{k-1}})$ is computed at a given confidence level, say 99%. Since the probability of default follows a sqare root means reverting process, at time t the distribution at a future time t' of the different levels of the default intensity λ_t is known to be a non-central χ^2 distribution.[6] This allows to compute, at a given date, which is the maximum level (with a predefined confidence level) of the default intensity λ_t and hence the maximum level of the spread and of the total cost for the refunding of each funding source. Besides, we want that the expected level of the spread is the forward spread implied by the curve referring spread discount bonds, that is for any $t < t' < T$:

$$P^s(0, T) = P^s(0, t')E^{t'}[P^s(t', T)]$$

which means that we want to compute the maximum level of the spread under the forward-risk adjusted measure.[7] We then need the forward-risk adjusted distribution of the default intensity, given in Cox, Ingersoll and Ross [4]:

$$p^{t'}_{\lambda_t}(\lambda_{t'}) = \chi^2\left(2\lambda_{t'}(\phi+\eta); \frac{4\kappa_\lambda\theta_\lambda}{\sigma^2_\lambda}, \frac{2\phi^2\lambda_t \exp[\gamma(t'-t)]}{\phi+\eta}\right)$$

where

$$\gamma = \sqrt{\kappa^2_\lambda + 2\sigma^2_\lambda}$$

$$\phi = \frac{2\gamma}{\sigma^2_\lambda(\exp[\gamma(t'-t)] - 1)}$$

$$\eta = \frac{\kappa_\lambda + \gamma}{\sigma^2_\lambda}$$

Assume we build a term structure of stressed spread discount bonds up to an expiry T_b. Assume also that a the roll-over of the debt occurs each of J years, so that it entails a number of refunding dates $\frac{(T_b - T_a)}{J} - 1 = n$. We run the following procedure described in a pseudo-code:

Procedure 5.1. *We first derive the maximum expected levels of the default intensity $\lambda^*_{t_i}$, at the scheduled refunding dates, with a confidence level cl (e.g.: 99%):*

1. ***For*** $i = 1, ..., n$
2. $T_i = i \cdot J$
3. $\lambda^*_{T_i} = \lambda_{T_i} : p^{T_i}_{\lambda_t}(\lambda_{T_i}) = cl$
4. ***Next***

---

[6] The non-central χ^2, with d degrees of freedom and non-centrality parameter c, is defined as the function $\chi^2(x; d, c)$.

[7] The superscript t' to the expectation operator $E[\cdot]$ means that we are working in the t'-forward-risk adjusted measure. Technically speaking, we are calculating expectations by using the bond $P^s(0, t')$ as a numeraire.

Once determined the maximum default intensity's levels, we can compute the term structure of (minimum) discount factors for the zero-spreads corresponding to those levels:

1. ***For*** $i = 1, ..., n$
2. $\quad T_i = i \cdots J$
3. $\quad$ ***For*** $k = 1, ..., J$
4. $\quad\quad P^{s^*}(0, T_{i+k}) = P^s_{cl}(0, T_{i+k}) = P^s(0, T_i)P^s(T_i, T_k; \lambda^*_{T_i}, \kappa_\lambda, \theta_\lambda, \sigma_\lambda, \mathcal{R}^J)$
5. $\quad$ ***Next***
6. ***Next***

Having the minimum discount factors for each expiry, we can compute the total minimum discount factor for all the expiries as:

$$P^D_{cl}(0, T_i) = P(0, T_i)P^s_{cl}(0, T_i) = P(0, T_i)P^{s^*}(0, T_i) \tag{28}$$

for $i = 1, ..., N$.

In building such curves we considered that during the period between two refunding dates, the cost of funding is completely determined by the maximum $\lambda^*_{T_i}$ *at the beginning of the same period. In fact we do not have any refunding risk and the curve is as it were derived with deterministic spreads.*

The unexpected funding cost in (27), at a given confidence level, can be now readily computed for each period. To cover these unexpected costs the bank posts economic capital. At time T_{d_k} the posted capital is:

$$E(T_{d_k}) = \sum_{m=k+1}^{b^*-k-1} \mathbf{UFC}(T_{d_m}) \tag{29}$$

$b^* \leq b$ is the number of periods that the financial institution deems reasonable to recapitalize the firm, should unexpected economic losses occur. The safest assumption is to set $b^* = b$, so that the full economic capital needed up to the expiry of the swap is taken into account. It is also true that usually market VaR is typically computed for a period of 1 year in banks, so that different choices can be adopted.

Assuming that required economic capital is invested in risk-free assets, the annual premium rate π over the risk-free rate to remunerate it,[8] is a cost that the bank has to bear to cover unexpected funding costs. For simplicity's sake, without too much loss of generality, let π be a constant; we have that the total funding cost is given by the amount of the debt left unpaid at the end of the swap, plus the present value of the annual premium paid on the economic capital for each period:

$$\mathbf{FC} = P(t, T_b)\mathbf{EFC}(T_b) + \sum_{k=1}^{M} P(t, T_{d_{k-1}})\pi E(T_{d_{k-1}})\delta_k \tag{30}$$

As above, the quantity **FC** is plugged in (16) to derive the fair swap rate, via a numerical search. The rate is once again the level making nil the present value of the funding cost $\mathbf{FC} = 0$.

[8] Basically the ROE deducted the risk-free rate.

6 Practical Examples

We show below how to implement in practice the strategies we have described above. We will price a market standard 10-years swap, with the fixed leg paying annually and the floating rate paying semi-annually: both legs have a fixed notional amount equal to 100. To value the fair rate of this swap, without including any other adjustment due to counterparty risk and funding costs, we need the term structure of OIS and 6M Libor, from which we derive also the discount factors. We adopt the market practice to consider the OIS the best proxy for the risk-free rate in the interbank market. Table 1 shows these data.

Year	**Eonia Fwd**	**6M Fwd Libor**	**OIS DF ($P(0,T)$)**	**Libor DF**
0	0.75%	1.40%	1.00000	1.00000
0.5	0.75%	1.39%	0.99626	0.99305
1	1.75%	2.39%	0.99254	0.98618
1.5	2.00%	2.63%	0.98393	0.97454
2	2.25%	2.88%	0.97419	0.96189
2.5	2.37%	2.99%	0.96335	0.94826
3	2.50%	3.11%	0.95207	0.93430
3.5	2.65%	3.26%	0.94032	0.91998
4	2.75%	3.35%	0.92802	0.90523
4.5	2.87%	3.47%	0.91543	0.89031
5	3.00%	3.59%	0.90248	0.87514
5.5	3.10%	3.69%	0.88915	0.85971
6	3.20%	3.78%	0.87557	0.84415
6.5	3.30%	3.88%	0.86179	0.82849
7	3.40%	3.97%	0.84780	0.81274
7.5	3.50%	4.07%	0.83363	0.79692
8	3.60%	4.16%	0.81929	0.78105
8.5	3.67%	4.23%	0.80480	0.76513
9	3.75%	4.30%	0.79030	0.74930
9.5	3.82%	4.37%	0.77575	0.73353
10	3.90%	4.44%	0.76121	0.71786

Table 1: Term structures of OIS and 6M Libor forward rates and of the corresponding discount factors for both.

The funding costs that the bank has to pay depend on the probability of default modelled in the reduced form setting with a stochastic intensity whose parameters are shown in Table 2. The resulting spread discount bonds and the total discount factors are in Table 3, where also forward funding spreads, defined as in (20), are shown.

λ_0	0.50%
κ_λ	1.00
θ_λ	1.95%
σ_λ	20.00%
$\mathcal{R}$	0%

Table 2: Parameters of the default intensity.

Given the market data above, the fair swap rate can be easily derived and it is $S_{0,10}(0) = 3.3020\%$. The future cash-flows of this swap can be hedged, as suggested above, with a portfolio of 1-year 1-year forward starting swaps (except the first one that is a 1-years spot starting swap); these swaps have to be market standard, in the sense that the fixed leg pays annualy whereas the floating leg pays semi-annually, similarly to the

Year	$P^s(0,T)$	$P^D(0,T)$	**Fwd Funding Spread**
0	1.00000	1.00000	
0.5	0.99597	0.99225	0.81%
1	0.98975	0.98237	1.26%
1.5	0.98226	0.96647	1.53%
2	0.97405	0.94891	1.69%
2.5	0.96545	0.93007	1.78%
3	0.95666	0.91081	1.84%
3.5	0.94779	0.89122	1.87%
4	0.93891	0.87132	1.89%
4.5	0.93005	0.85140	1.90%
5	0.92125	0.83141	1.91%
5.5	0.91251	0.81135	1.92%
6	0.90384	0.79138	1.92%
6.5	0.89525	0.77151	1.92%
7	0.88674	0.75177	1.92%
7.5	0.87830	0.73217	1.92%
8	0.86994	0.71273	1.92%
8.5	0.86167	0.69347	1.92%
9	0.85347	0.67449	1.92%
9.5	0.84534	0.65578	1.92%
10	0.83730	0.63736	1.92%

Table 3: Term structures of spread and total discount factors and forward funding spreads.

10-year swap. In Table 4 we show the fair swap rate for each hedging swap, for the year when the correspondigf fixed leg pays. The floating leg of each hedging swap matches a portion on the floating leg of the 10-year swap. Assuming that the bank is receiver fixed rate on the 10-year swap, net cash-flows for the hedged position are shwon in Table 4. In Figure 1 we show the cumulated cash-flows, whose value, compunded at the risk-free rate, sums algebrically up, obviously, to zero.

From Table 4 one can check that the receiver swap, once hedged, does not imply any negative cumulated cash flow, so that the bank does not have to resort to any additional external funding. The fair swap rate is for the bank the same calculated above and no adjustments for funding costs need to be included. This does not mean that the **CVA** for the counterparty credit risk and the **DVA** for its own default risk does not have to be considered, although we do not do so in the current analysis: this example demonstrates that the **DVA** is not the funding cost for a derivative contract, in accordance with Castagna [3].

Assume now that the bank has a payer receiver in the 10-year swap: all cash-flows with a positive (negative) sign in table 4 should now be considered as paid (received), so that the compounded cumulated cash flow is always negative and nil at expiry. This is true if the bank is able to borrow money at the risk-free rate; since the bank can actually go defaulted with a positive probability, it pays a funding spread to borrow money. We analyse both strategies suggested above to cope with funding needs originated by the negative cumulated cash-flows and verify how the fair swap rate is modified.

Let us start with the Strategy 1, or funding all negative cash-flows at inception. The numerical search of the starting amount of the debt, subject the the constraints stated above, and of the fair swap rate that makes nil the present value of the funding cost **FC**, are shown in Table 5. The fixed interest rate paid annually by the bank on the debt is 4.2761% and it is obtained via (14). This rate applied to the debt oustanding at the beginning of the period yields the interests paid. The starting amount A that the bank has to borrow is 4.1746 and the amortization plan shown guarantees that is fully repaid

Year	Hdg Swaps	Cash-flows	Cumulated cash-flows	Compounded Cum. cash-flows
0				
0.5				
1	1.40%	1.902593	1.9026	1.9026
1.5				
2	2.52%	0.780902	2.6835	2.7193
2.5				
3	2.95%	0.352942	3.0364	3.1355
3.5				
4	3.21%	0.096033	3.1325	3.3127
4.5				
5	3.43%	-0.13114	3.0013	3.2753
5.5				
6	3.67%	-0.36408	2.6373	3.0119
6.5				
7	3.86%	-0.55698	2.0803	2.5536
7.5				
8	4.05%	-0.75044	1.3298	1.8921
8.5				
9	4.23%	-0.92912	0.4007	1.0323
9.5				
10	4.37%	-1.07261	-0.6719	0.0000

Table 4: Swap rates of the hedging swaps and net single, cumulated and compounded cumulated cash-flows for a hedged 10-year receiver swap.

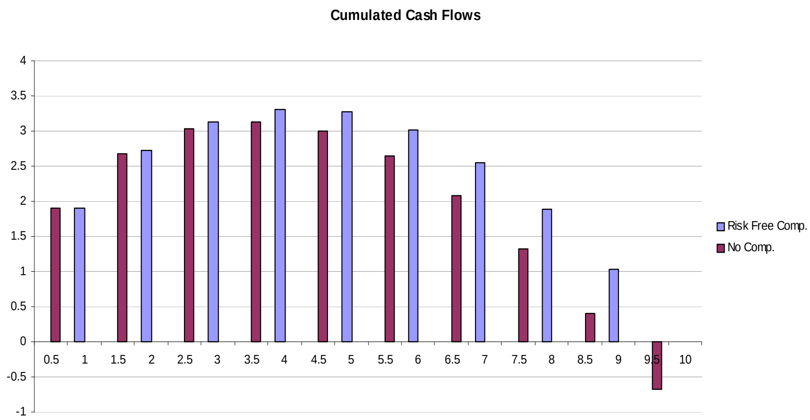

Figure 1: Compounded and non compounded cumulated cash-flows for a 10-year receiver swap.

and that no available liquidity is left at the expiry of the contract. The final fair swap rate is $S^{\mathbf{FC}}_{0,10}(0) = 3.2403\%$, a correction of around 6 bps.

Year	Outstanding Debt $A(T_{d_k})$	Interests Paid	Available Liquidity $\mathbf{AVL}(T_{d_k})$
0	4.1746		
0.5	4.1746		
1	4.1746	0.1785	2.1866
1.5	4.1746		2.2057
2	4.1746	0.1785	1.3300
2.5	4.1746		1.3450
3	4.1746	0.1785	0.8912
3.5	4.1746		0.9023
4	4.1746	0.1785	0.7014
4.5	4.1746		0.7111
5	3.9817	0.1785	0.5428
5.5	3.9817		0.5509
6	3.5560	0.1703	0.3892
6.5	3.5560		0.3954
7	2.9373	0.1521	0.2499
7.5	2.9373		0.2541
8	2.1252	0.1256	0.1330
8.5	2.1252		0.1354
9	1.1343	0.0909	0.0470
9.5	1.1343		0.0479
10	0.0000	0.0485	0.0000

Table 5: Strategy 1: Amount of the outstanding debt, interest paid and available liquidity. Final values maybe slightly different from zero due to the degree of approximation chosen in the numerical search.

Let us examine now how Strategy 2 can be implemented: the bank borrows money when negative cash-flows occur, if cumulated cash-flows are negative, and the debt is rolled over in the future. The unexpected funding cost is measured in the first of the two approaches proposed, that is by means of spread options. In Table 6 the results are shown. The terminal outstanding debt is negative (i.e.: there is a cash inflow) and its present value compensates the sum of the present value of unexpected funding costs (last column), $\sum_{k=1}^{M} P(t, T_{d_{k-1}})\mathbf{UFC}(T_{d_k}) = 0.0881$; the final fair swap rate is $S^{\mathbf{FC}}_{0,10}(0) = 3.2493\%$.

In Table 7 we present results if the second approach is adopted to measure unexpected funding costs. The spread discount factors at a confidence level of 99% are computed with the procedure outlined above, and they are shown in the last column. We assume a constant premium over the risk-free rate for the economi capital equal to $\pi = 5\%$. The capital is posted to cover at any time all future losses until the expiry of the contract, so that $b^* = b$ in formula (29). The fair swap rate is once again computed so that the total funding cost is nil and it is $S^{\mathbf{FC}}_{0,10}(0) = 3.2089\%$. The terminal oustanding amount of the debt is negative, meaning that the bank has an inflow: also in this case, the present value of this positive cash flow compensates the cost of the economic capital posted to cover unexpected funding losses, $\sum_{k=1}^{M} P(t, T_{d_{k-1}})\pi E(T_{d_{k-1}})\delta_k = 0.47889$.

Finally, in Table 8 we summarize results to allow for an easy comparison amongst the different way to include the funding costs in the pricing of a swap. Given the term structure of interest rates and of probability of default, the Strategy 1 (funding everything at inception) and the Strategy 2, with unexpected finding costs measured with spread options, produce very similar results: the fair rate of a payer swap is abated by about 6 bps in both cases. The Strategy 2, with unexpected costs measured at a given confidence level and covered with economic capital, is more expensive and the fair swap rate is decreased

Year	cash-flows Paid	Cumulated cash-flows	Compounded cash-flows	Debt Roll-Over $\mathbf{FDB}(T_k)$	Unexp'ed FC $P(t, T_{d_{k-1}})\mathbf{UFC}(T_{d_k})$
0					
0.5					
1	1.8498	1.8498	1.8498	1.8498	0.0062
1.5					
2	0.7282	2.5780	2.6128	2.6432	0.0107
2.5					
3	0.3002	2.8782	2.9737	3.0540	0.0128
3.5					
4	0.0433	2.9215	3.0941	3.2357	0.0136
4.5					
5	-0.1839	2.7376	2.9978	3.2071	0.0132
5.5					
6	-0.4168	2.3208	2.6731	2.9525	0.0118
6.5					
7	-0.6097	1.7110	2.1509	2.4983	0.0097
7.5					
8	-0.8032	0.9079	1.4226	1.8320	0.0069
8.5					
9	-0.9819	-0.0740	0.4929	0.9540	0.0034
9.5					
10	-1.1254	-1.1994	-0.6136	-0.1158	

Table 6: Strategy 2, first approach: Single and cumulated cash-flows, debt roll-over and present value of unexpected funding cost for each period measured with spread options. Final values maybe slightly different from zero due to the degree of approximation chosen in the numerical search.

Year	cash-flows Paid	Debt Roll-Over $\mathbf{FDB}(T_k)$	Unexp.ed Cost $\mathbf{UFC}(T_{d_k})$	Posted EC $E(T_{d_k})$	EC Remun. $P(t, T_{d_{k-1}})\pi E(T_{d_{k-1}})\delta_k$	99% cl $P^{s^*}(T_{d_k})$
0					1.3983	1.00000
0.5						0.99118
1	1.8498	1.8095	1.8095	0.0000	1.3983	0.98237
1.5						0.96129
2	0.7282	2.5611	2.5785	0.0173	1.3809	0.94021
2.5						0.91692
3	0.3002	2.9281	2.9728	0.0446	1.3363	0.89362
3.5						0.86991
4	0.0433	3.0638	3.1424	0.0786	1.2578	0.84619
4.5						0.82264
5	-0.1839	2.9867	3.1033	0.1167	1.1411	0.79910
5.5						0.77591
6	-0.4168	2.6806	2.8373	0.1568	0.9843	0.75273
6.5						0.73018
7	-0.6097	2.1718	2.3681	0.1964	0.7879	0.70763
7.5						0.68577
8	-0.8032	1.4472	1.6806	0.2334	0.5546	0.66391
8.5						0.64283
9	-0.9819	0.5070	0.7723	0.2653	0.2893	0.62175
9.5						0.60158
10	-1.1254	-0.6291	-0.3398	0.2893		0.58142

Table 7: Strategy 2, second approach: Single and cumulated cash-flows, debt roll-over and present value of unexpected funding cost for each period, measured at a confidence level of 99%. Final values maybe slightly different from zero due to the degree of approximation chosen in the numerical search.

by around 10 bps.

It is worth noticing that this relationship amongst the three adjustments may not hold in every case. It may well be the case that for forward starting swaps, say a 10Y5Y, the Strategy 2, first approach, may result more convenient than Strategy 1. In any case, the only hedging scheme fully protecting the bank is Strategy 1, since it avoids also the exposure to future liquidity shortages, so that one should consider also this risk, which is very difficult to measure.

7 Conclusions

We have shown in this paper how to include the funding costs in the pricing of interest rate swaps. We proposed two strategies (and two versions for the second) to account in a consistent and thorough fashion for the funding spread that a bank has to pay when borrowing money. The outlined methods clearly show that for interest rate swaps the funding costs is not related at all at the **DVA**, which also depends on the probability of default of the bank, but has a different nature, as we proved elsewhere.

Future research should consider the effects of the counterparty's default on the funding strategies, and how funding costs can be included in the pricing of collateralised swaps. An interesting area is also the inclusion of funding costs in CDSs.

	Fair Swap Rate	**FVA**
Pure Rate	3.3020%	
With FC Strategy 1	3.2403%	0.5463
With FC Strategy 2		
First Approach UFC	3.2493%	0.4667
Second Approach UFC	3.2089%	0.8240

Table 8: Effects on the fair swap rate of the inclusion of funding costs according to the different methods proposed.

References

[1] B. Bianchetti. Two curves, one price: Pricing and hedging interest rate derivatives decoupling forwarding and discounting yield curves. *Working Paper. Available at http://papers.ssrn.com*, 2008.

[2] D. Brigo and A. Capponi. Bilateral counterparty risk valuation with stochastic dynamical models and applications to credit default swap. *Risk*, March, 2010.

[3] A. Castagna. Funding, liquidity, credit and counterparty risk: Links and implications. *Iason research paper. Available at http://iasonltd.com/resources.php*, 2011.

[4] J. C. Cox, J. E. Ingersoll, and S. A. Ross. A theory of the term structure of interest rates. *Econometrica*, 53:385–467, 1985.

[5] D. Duffie and M. Singleton. Modeling term structure of defaultable bonds. *Review of Financial Studies*, (12), 1999.

[6] F. Mercurio. Interest rates and the credit crunch: New formulas and market models. *Working Paper. Available at http://papers.ssrn.com*, 2010.

[7] P. J. Schonbucher. A libor market model with default risk. *Working Paper. Available at http://papers.ssrn.com*, 2000.