Sight Deposit and Non-Maturing Liability Modelling

In this article we present a review of the most significant approaches provided by the literature and the market practice for the modeling of non-maturing deposits accounts. We describe the bond portfolio replication approach and then move to the class of stochastic factor models, showing how the latter are capable of provide more effective tools for the interest rate and liquidity risk management of these balance-sheet items.

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THE modelling of deposits and non-maturing liabilities is a crucial task for the liquidity management of a financial institution. It has become even much more crucial in the current environment after the liquidity crisis that affected the money market in 2008/2009. Typically ALM departments of banks, involved in the management of interest rate and liquidity risks, face the task of forecasting deposit volumes, so as to design and implement consequent liquidity strategies. Moreover deposit accounts represent the main source of funding for the bank, primarily for those institutions focused on the retail business, and they heavily contribute to the funding available in every period for lending activity. Amongst the different funding sources, deposits have lower costs, so that in a *funding mix* they contribute to abate the total cost of funding.

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The optionality embedded in non-maturing products, related to the possibility for the customer to arbitrarily choose any desired schedule of principal cash-flows, has to be understood and accounted for when performing liabilities valuation and hedging the market and liquidity risk. Thus a sound model is essential to deal with nested optionality for liquidity risk management purposes.

Modelling Approaches

There are two different approaches in the financial literature and the market practice for the modelling of deposits' balance evolution:

- Bond portfolio replication,
- OAS models.

The bond portfolio replication, probably the most common approach adopted by banks, can be shortly described as follows. First, the total deposits' amount is split in two components:

- a *core* part that is assumed to be not sensible to market variable evolution, such as interest rates and deposit rates. This fraction of the total volume of deposits is supposed to decline gradually on a medium-long term period (say, 10 or 15 years) and to amortise completely at the end of it.
- a *volatile* part that is assumed to be withdrawn by depositors over a short horizon. This fraction basically refers to the component of the total volume of deposits that is normally used by depositors to match their liquidity needs.

Secondly, the core part is hedged with a portfolio of vanilla bonds and money market instruments, whose weights are computed by solving an optimisation problem that could be set according to different rules. Typically, the portfolio weights are chosen so as to replicate the amortisation schedule of deposits or, equivalently said, their duration. In this way the replication portfolio tries and preserve the economic value of the deposits (as defined later on) against the market interest rates' movements. Another constraint, usually imposed in the choice of portfolio weights, is the target return expressed as a certain margin over the market rates. Since deposit rates are updated, within a relatively large freedom of action, by banks to align them to market rates, the replication portfolio can comprise fixed rate bonds, to match the inelastic part of the deposit rates that is not reactive to changes of market rates, and floating rate bonds, to match the elastic part of the deposits rates. The process to re-balance the bond portfolio, although simple in theory, is quite convoluted in practice. For an more detailed explanation of the mechanism, see Bardenhewer in [8].

Thirdly, the volatile part is invested in very short term assets, typically in overnight deposits, and it represents a liquidity buffer to cope with the daily withdrawals by depositors.

The critical point of this approach stands in the estimation of the amortisation schedule of non-maturing accounts, that is performed on statistical bases and have to be reconsidered periodically. One of the flaws of the bond replica approach is that the risk-factors affecting the evolution of the deposits are not modelled as stochastic variables. As such, once the statistical analysis is performed, the weights are applied by considering the current market value of the relevant factors (basically, market and deposit rates) without considering their future evolution. This flaw is removed, at least partially, by the so called Option Adjusted Spread (OAS) approach, which we prefer to define as Stochastic Factor (SF) approach.[1] The approach is not in principle different from the Bond Portfolio Replica approach: one tries and identify statistically how the evolution of the deposit's volume is linked to risk factors (typically market and interest rates) and then set up a hedge portfolio that covers the exposures them.

The main difference lies in that, differently from the Bond Portfolio Replica, in the SF approach the weights of the hedging instruments are computed considering the future random evolution of the risk factors, so that the hedging activity resembles the dynamic replication of derivatives contracts. The hedging portfolio is revised based on the market movements of the risk factors, depending on the stochastic process adopted to model them. We prefer to work with a SF approach to model deposit volumes for several reasons. First, we think that the SF approach is more advanced under a modelling perspective, taking into account explicitly the stochastic nature of the risk factors. Secondly, if the Bond Portfolio Replica can be deemed adequate for hedging the interest rate margin and the economic value of the deposits, under a liquidity risk management point of view the SF approach is superior, for the very fact that it is possible to jointly evaluate within a unified consistent framework the effects of the risk factors both on the economic value and on the future inflows and outflows of the deposits. Thirdly, it is easier to include in the SF approach complex behavioural functions linking the evolution of the volumes to the risk-factors. Finally, bank-run events can be also considered and prop-

[1] We think that OAS is misleading for a number of reasons: the approach does not explicitly model any optionality and does not adjust any spread, as it will be clear from what we will show below. The name is likely derived from a contagion from the (bad) practice, in the fixed income market, to use an effective discount rate to price assets taking into account embedded optionalities (whence the name).

erly taken into account in SF approach, whereas it seems quite difficult their inclusion within a Bond Portfolio Replica approach.

The Stochastic Factor Approach

The first attempt to apply the SF approach, within an arbitrage-free derivatives pricing framework, to deposit accounts was made by Jarrow and van Deventer [1]. They derived a valuation framework for deposits based on the analogy between these liabilities and an exotic swap whose principal depends on the past history of market rates. They provide a linear specification for deposit volumes evolution applied to U.S. federal data.
Other similar models have been proposed,[2] within the SF approach: it is possible to identify three building blocks common to all of them:

1. a stochastic process for the interest rates: in the above mentioned Jarrow and van Deventer, for example, it is the Vasicek model;

2. a stochastic model for the deposit rates: typically these are linked to the interest rates by means of a more or less complex function;

3. a model for the evolution of the volume of deposits: since this is linked by some functional forms to the two risk factors at points 1 and 2, it is a stochastic process as well.

The specification of deposit volumes dynamics is the crucial feature distinguishing the different SF models: looking things under a micro-economic perspective, volumes depend on the liquidity preference and risk-aversion of depositors, whose behaviour is driven by the opportunity costs between alternative allocations. When market rates rise, depositors have a greater convenience to withdraw money from sight deposits and invest it in other assets offered in the market. SF models can be defined *behavioural* in the sense that they try to capture the dynamics of depositors's behaviour with respect to market rates and deposit rates movements. In doing this, these models exploit the option pricing technology, developed since 1970s, and depend on stochastic variables, in contrast with the previously mentioned class on simpler statistical models, as mentioned above. Depositors' behaviour is synthesized in a behavioural function that depends on risk factors and determines their choice in terms of amount allocated in deposits. This function could be specified in various forms, allowing for different degrees of complexity. Given their stochastic nature, those models are suitable to be implemented in simulation-based framework like Montecarlo methods. Since closed-form formulae for deposits' value are expressed as risk-neutral expectations, the scenario generation process has to be accomplished with respect to the equivalent martingale probability measure. For liquidity management purposes, it is more appropriate to use real-world parameter processes. In what follows we will not make any difference, though: as we have assumed also in other parts of this book, assuming a risk-aversion parameter equal to zero, real-world process for interest rates clash with risk-neutral ones. We propose a specification of the SF approach that we think is enough parsimonious, yet effective.

Modelling of Market Interest Rates

The dynamics for the market interest rates can be chosen rather arbitrarily in the class of short rate models. In the our specification we adopted a one-factor CIR++ model: we know that such model is capable to to perfectly match the current observed term structure of risk-free zero rates. The market instantaneous risk-free rate is thus given by

$$r_t = x_t + \phi_t$$

where $x_t$has dynamics

$$dx_t = k\left(\theta - x_t\right)dt + \sigma\sqrt{x_t}dW_t$$

and $\phi_t$is a deterministic function of time.

Modelling of Deposit Rates

The deposit rate evolution is linked to the pricing policy of the banks, providing a tool that can be exploited to drive deposits volume across time. It is reasonable to think that an increase of the deposit rate will work as an incentive for existing depositors not to withdraw from their accounts or to even increase the amount deposited. The rate paid by the bank on deposit accounts can be determined according to different rules. Here are some examples:

1. constant spread below market rates:
$$d_t = \max\left[r_t - \alpha, 0\right]$$
to avoid having negative rates on the deposit, there is a floor at zero.

2. a proportion $\alpha$of market rates:
$$d_t = \alpha r_t$$

3. a function similar to the two above but dependent also on the amount deposited:
$$d_t = \sum_{j=1}^{m} i_j(r_t)\,\mathbf{1}_{\{D_t^j, D_t^{j+1}\}} D_t$$

[2]See, amongst others, Frauendorfer and Schürle in [2], Dewachter *et al.* , Kalkbrener and Willing [9] and Blöchlinger [4]

where $D^j$and $D^{j+1}$are range of the deposit volume D producing different level of the deposit's rate.

We adopt a rule slightly more general than the proportional one, *i.e.*: a linear affine relation between the deposit rate and the market short rate

$$d_t = \alpha + \beta r_t + u_t \quad (1)$$

where $E(u_t) = 0 \quad \forall t$. As it will be manifest in what follows, the deposit volume's evolution depends on the deposit rate, so in this framework the pricing policy function, that is obviously discretionary for the bank, represents a tool to drive deposit volumes and consequently it can be used to define liquidity strategies.

Modelling of Deposit Volumes: Linear Behavioural Functions

We can model the evolution of the total deposit volume by establishing a linear relationship between its log-variations and the risk factors, *i.e.*: the market interest and deposit rates: this is the simplest behavioural functional form we can devise. Moreover we add an autoregressive component, by imposing that the log-variation of the volume at a given time is linked to the log-variation of the previous period with a given factor and finally we include also a relationship with the time, so as to detect time-trends. The volume's evolution is in this case given by the equation:

$$\begin{aligned}\log D_t = \gamma_0 + \gamma_1 \log D_{t-1} + \gamma_2 t \\ + \gamma_3 \Delta r_t + \gamma_4 \Delta d_t + \epsilon_t\end{aligned} \quad (2)$$

with Δ being the first-order difference operator and ϵ_t the idiosyncratic error term with zero mean. This formula is in practice the same as in Jarrow and van Deventer.

The model in 2 is convenient because parameters can be easily estimated on historical data via standard OLS algorithm. The presence of a time component in equation 2 is justified by empirical evidence on deposit series, that exhibit a trend component. This factor could be modelled in alternative ways, substituting the linear trend with a quadratic or exponential one. For interest rate risk management purposes, we can however be interested in understanding how deposit evolution explained only by market and deposit rates' movements. To this end, we can introduce a reduced version of the model that is estimated excluding the trend component, *i.e.*:

$$\log D_t = \gamma_0 + \gamma_1 \log D_t + \gamma_2 \Delta r_t + \gamma_3 \Delta d_t \quad (3)$$

Empirical analysis of both the model's form will be presented below.

Modelling of Deposit Volumes: Non-Linear Behavioural Models

The behavioural function linking the evolutions of deposits' volume to the risk factors can be also non-linear, possibly involving complex forms. In recent years some efforts have been made to formulate this relation according to more sophisticated functions, trying to describe peculiar features of deposits's dynamics. The main contribution in this direction was provided by Nyström [6], who introduced in the valuation SF framework we are discussing a non-linear dependency of the deposit volumes dynamics from the interest rates. The formalization of such dynamical behaviour is not trivial and we propose a model specification, inspired to the cited Nyström's work. The main reason why non-linear behavioural functions have been proposed is a drawback of equation 2: it does not allow to fully capture the empirically observed depositors' reactions to market and deposit rates' movements. Actual behaviour exhibits high non-linearity with respect to these, in the sense that it depends not only on variations, as implied by equation 2, but also on the levels of market and deposit rates.
The main idea in modelling the non-linear behaviour is based on the micro-economic liquidity preference theory: depositors (and, generally speaking, investors) prefer to keep their investments in liquidity for low levels of market rates. As market rates increase, the preference for liquidity is counterbalanced so that depositors transfer higher fractions of their income and wealth to less liquid investments. In more detail, the first variable to consider is the total depositors' income, I, growing at an annual rate ρ: on an aggregated base we could see it as the growth rate of the economy (GDP) or simply the growth rate of the income for each depositor (customer). Secondly, the allocation of the income between deposits and other (less liquid) investments hinges on the following assumptions:

- each depositor modifies his balance on the deposit account targeting a given fraction $\overline{\lambda}$ of their income I. This level can be interpreted as the amount they need to cover his short-time liquidity needs. At any time t, given the current fraction λ_t of the income invested in deposit, the adjustment toward the target $\overline{\lambda}$ occurs at a speed ζ;

- there is an interest rate's strike level E, specific to the customer, such that, when the market rate is above it, then they reconsider the target level and redirects a higher amount to other investments, by a fraction γ of their income;

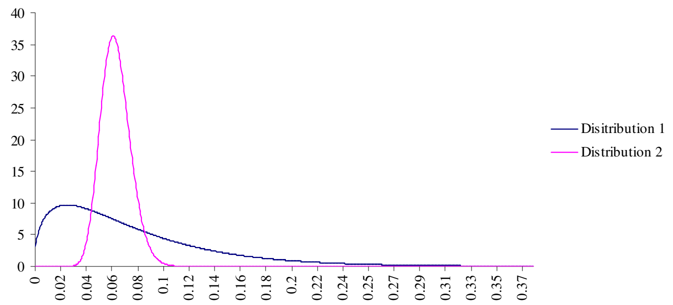

FIGURE 1: Two possible distributions produced by the Gamma function. On the x-axis: the interest rate level.

- there is an deposit rate's strike level F, specific to the customer, such that, when the rate received on deposits is above it, then they are more reluctant to withdraw money by a fraction δ of their income;

Under these assumptions, the evolution of the fraction λ_t of the income allocated in sight deposit is:

$$\lambda_{t+\Delta t} - \lambda_t = \zeta(\overline{\lambda} - \lambda_t)\Delta t + \gamma \mathbf{1}_{[E,\infty)}(r_t) + \delta \mathbf{1}_{[F,\infty)}(r_t) \quad (4)$$

where $1_{[E,\infty)}$ is the indicator function equal to 1 when the condition in the subscript is verified. The income I grows as follows:

$$I_{t+\Delta t} - I_t = I_t \rho \Delta t \quad (5)$$

and the deposit volume at time t is:

$$D_t = \lambda_t I_t \quad (6)$$

In reality, since each depositor has different level of strike rates E and F, due to their preferences for liquidity, on an aggregated basis, considering all bank's customers, there is a distribution of strike rates, reflecting their heterogeneity in behaviour. So, when we pass from the evolution of the single deposit, to the evolution of the total volume of deposits on bank's balance sheet, strike rates can be thought to be distributed according to any suitable probability function $h(x)$: in the specification we present here we choose a Gamma function, *i.e.*:

$$h(x;\alpha,\beta) = \frac{(x/\beta)^{\beta-1}\exp(-x/\beta)}{\beta\Gamma(\alpha)}$$

EXAMPLE 1 The Gamma function is very flexible and it allows for a wide range of possible shapes of the distribution. If we set $\alpha = 1.5$ and $\beta = 0.05$, or example, we have a distribution labeled as "1" in Figure 1. If $\alpha = 30$ and $\beta = 0.002$ we have a distribution "2". It is possible to model the aggregated customers' behaviour, making it more or less concentrated around specific levels.

We can alternatively use the equivalent functional form of the Gamma distribution written as:

$$h(x;k,\theta) := \frac{1}{\theta^k \Gamma(k)} t^{k-1} e^{-\frac{t}{\theta}}$$

This is actually what we will use in the estimation of the parameters from historical data we will show below.

The evolution of total volume of deposits can be written by modifying equation 4 and considering the distributions of the strike rates instead of the single strike rates for each depositor:

$$\lambda_{t+\Delta t} - \lambda_t = \zeta(\overline{\lambda} - \lambda_t)\Delta t + \gamma H(r_t, k_1, \theta_1) + \delta H(d_t, k_2, \theta_2) \quad (7)$$

where $H(x,k,\theta) = \int_0^x h(u;k,\theta)du$ is the Gamma cumulative distribution function.

To make the econometric estimation of the parameters easier, we rewrite equation 7 in the following way:

$$\lambda_t = \alpha + \beta\lambda_{t-1} + \gamma H(r_t, k_1, \theta_1) + \delta H(d_t, k_2, \theta_2) \quad (8)$$

where $\alpha = \zeta\overline{\lambda}\Delta t$ and $\beta = \zeta\Delta t$. Equation 8 can be applied by the bank to the "average customer". Given the heterogeneity of the behaviours, given current market and deposit rates, the incentive to change the income allocation by increasing less liquid investments, balanced by the incentive to keep the investment in deposits provided by the deposit rates, is synthesized in the Gamma distribution functions, so that $H(x,k,\theta)$ turns out to be the cumulative density of the average customer's strike.

Economic Evaluation and Risk Management of Deposits

The three building blocks to model the deposits can be used to compute the economic value to the bank of the total amount held on balance sheet. At time

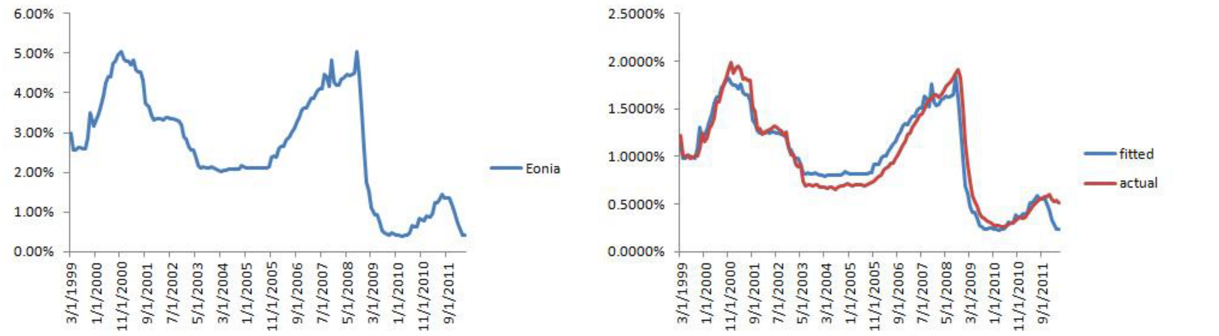

FIGURE 2: On the left: time series of 1-month Eonia swap rates for the period 3/1999:4/2012; on the right: actual time series of deposit rates vs. fitted values.

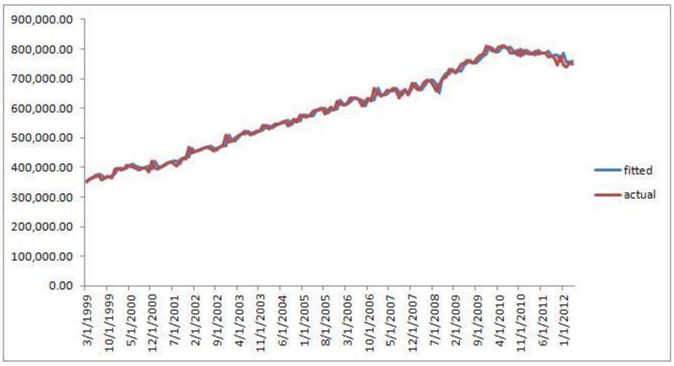

FIGURE 3: Actual time series of deposit volumes vs. fitted values for the linear behavioural model.

$t = 0$, for a time horizon T, the economic value is the expected margin that can be earned by the bank on the present and future volume of deposits. In fact, the amount of funds raised by the bank in form of deposits can be invested at in short expiry risk-free investments yielding r_t; on the other hand the deposits cost to the bank the rate d_t that it has to pay to depositors. In formula:

$$V^D(0,T) = \sum_{j=1}^{n} \int_0^T E^Q \left[\left(r_t - d_{j,t} \right) D_{j,t} P^D(0,t) \right] dt \tag{9}$$

where $D_{j,t}$ is the amount deposited in the account j at time t and n is the number of deposit accounts. The expectation is taken under the equivalent martingale risk-neutral measure Q. Equation 9 is the expected Net Interest Margin to the bank over the period $[0,T]$, for all the deposits accounts, discounted in 0 by the risk-free discount factor P^D. As suggested by Jarrow and van Deventer [1], the value of deposits can be seen as the value of an exotic swap, paying the floating rate $d_{j,t}$ and receiving the floating rate r_t, on the stochastic principal $D_{j,t}$ for the period between 0 and T. The approach we outlined above is also a good tool for liquidity risk management, since it can be used to predict expected or stressed (at a given confidence level) evolution of deposit volume. To compute these metrics, we need to launch a Montecarlo simulation on the two risk factors, *i.e.*: the risk-free instantaneous interest rate and deposit rate. We operate the following steps:

- given a time-horizon T, divide the period $[0,T]$ in M steps,;
- simulate N paths for each risk factor;
- compute the expected level of deposit volume $V(0,T_i)$ at each step $i \in \{0,1,...,M\}$, by averaging out on the N scenarios, by means of equation 2 or 8:

$$D^e(T_i) = E\left[D(T_i)\right] = \frac{\sum_{m=1}^{M} D^m(T_i)}{M}$$

- compute the stressed level of deposit volume at a given confidence level p, $V^p(0,T_i)$ at each step at each step $i \in \{0,1,...,M\}$, based in the M scenarios. Since for liquidity risk management purposes the bank is interested at the minimum levels of the deposit volume at a given time T_i, then we define the stressed level at p confidence level as:

$$D^p(T_i) = \inf\left\{D(T_i) : \Pr[D(T_i) < D^p(T_i)] \geq p\right\}$$

Banks can be interested in computing the minimum level of deposits during the entire period included between the reference time (say, 0) and a given time T_i: this is actually the value that corresponds to the actual available liquidity that can be used for investments expiring in T_i. To this end it is useful to introduce the process of the minima of the deposit volume, defined as:

$$D^{\min}(T_i) = \min_{0 \le s \le T_i} D(s)$$

Basically the process exclude all the growth of the volume of deposits due to new deposits or to an increase of the amount of the exiting ones, but it considers only the abating effects that the risk factors produce. The metric is also consistent with the factual truth that in any case the bank can never invest more than the existing amount of deposits it has on its balance sheet. The SF approach can be used also for interest rate management purposes. Once we have the computed the economic value of deposits, it is straightforward to compute its sensitivities to risk-factors to set up hedging strategies with liquid market instruments such as swaps. To this end, we can calculate the sensitivities of deposits' economic value to perturbations in the market zero-rate curve. The sensitivity to the forward rate $F(0; t_i, t_{i+1}) = F_i(0)$ is obtained numerically by means of the following:

$$\Delta V(0, T; F_i(0)) = V\left(0, T; \tilde{F}_i(0)\right) - V(0, T; F_i(0)) \tag{10}$$

where $V(\cdot)$is provided by (9) and $\widetilde{F}_i(0)$ is the relevant forward rate bumped by a given amount (*e.g.*: 10 bps). We have assumed that the instantaneous short rate follows a one-factor CIR++ dynamics. Assuming now that the initial zero-rate curve generated by the model, *i.e.*: the series $\left\{P^D(0, T_i)\right\}_{i=1}^{n}$, perfectly matches the market observed term structure, we have to modify the short rate dynamics in a way that produces the desired bump on the forward rates time 0, by suitably modifying the the deterministic time dependent term $\phi(t)$ of the CIR++ process. This is easily done: let bmp be the size of the bump to the term structure of starting forward rate $F_i(0)$; in the CIR++ the tilted forward $\tilde{F}_i(0)$ is obtained by modifying the integrated time dependent function $\phi(t)$ as:

$$\int_{T_i}^{T_{i+1}} \phi(s) ds \quad \rightarrow \quad \int_{T_i}^{T_{i+1}} \phi(s) + \frac{\ln(bmp)}{\tau_i} ds$$

where $\tau_i = T_{i+1} - T_i$. We present below some practical applications to the approach sketched above.

EXAMPLE 2

We perform an empirical estimation and test of the the SF approach, with the two behavioural functions we have presented above, based on public aggregated data for sight deposits in Italy. We considered a sample of monthly observations in the period 3/1999 : 4/2012 for sight deposits' total volume and average deposit rates paid by the bank. Data for deposits are published by Bank of Italy (Bollettino Statistico).[3] We considered the euro 1-month overnight index average (Eonia swap) rate as a proxy for the market short risk-free rate: values for the analysis period are plotted in Figure 2 on the left. The CIR model for the market rate was calibrated on the time series of Eonia rates via Kalman filter, and the resulting values for the parameters are:

$$\kappa = 0.053, \quad \theta = 7.3, \quad \sigma = 8.8\%$$

For the second building block (deposit rates), the linear relation between market rates and deposit rates in Equation 1 has been estimated via standard OLS, results are shown in Table 1. Figure 2 plots on the right the actual time series of deposit rates and fitted values from the estimated regression. The model shows a good fitting of the time series and we can observe that the linear affine relation is strongly consistent with the data.

	Coefficient	Significance (p-values)
Intercept α	1.05	0.042
Market rate r_t	0.92	1.86E-51
R^2	0.92	
F statistics	1773	
F significance	4.23E-87	

TABLE 1: Regression results for the deposit rate's equation 1.

	Coefficient	Significance (p-values)
Intercept	1.05	0.042
Lagged D_{t-1}	0.92	1.86E-51
Time t	0.4E-3	0.093
Market rate variations Δr_t	-3.45	0.001
Deposit rate variation Δd_t	7.54	0.009
R^2	0.99	
F statistics	4518	
F significance	1.17E-157	

TABLE 2: Regression results for the linear behavioural equation 2.

Finally, we need to adopt a behavioural function. We start with the linear model for deposit volumes in Equation 2: estimation results are shown in Table 2 and also in this case the model proves to be a good explanation of the data. We note that the signs of coefficients multiplying, respectively, the variations in the market rate and the variations in the deposit rate, are opposite as expected. Figure 3 plots actual and fitted time series of deposit volumes.

[3] Data are available also at the web site www.bancaditalia.it.

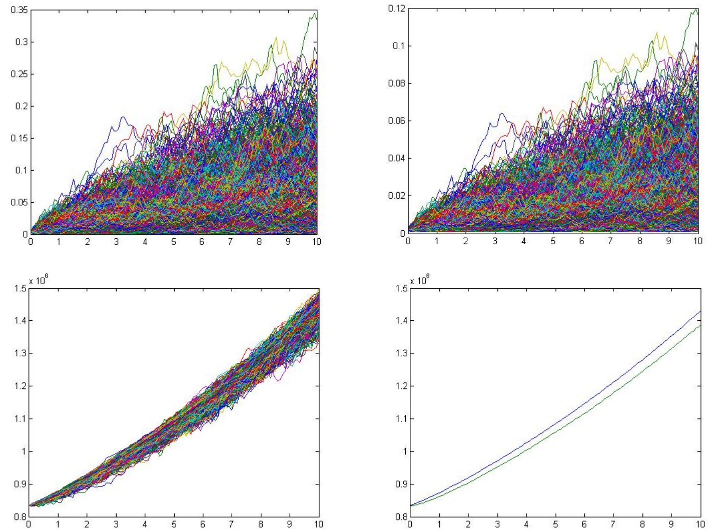

***FIGURE 4**: From the top on the left clockwise: simulated paths for 1-month Eonia swap rate, deposit rate and deposit volume, term structure of expected and minimum (99% c.l) future volumes.*

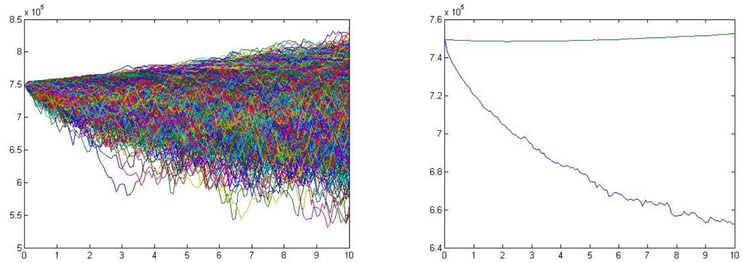

***FIGURE 5**: On the left, simulated paths for deposit volume; on the right: term structure of expected and minimum (99% c.l) future volumes.*

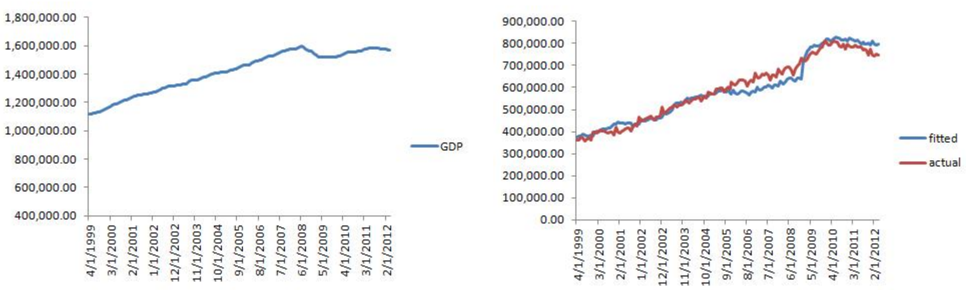

***FIGURE 6**: On the left: time series of Italian nominal GDP for the sample 3/1999:4/2012; quarterly data are linearly interpolated to obtain the monthly time series. On the right: Actual time series of deposit volumes vs. fitted values for the non-linear behavioural model.*

We can now use estimated parameters to compute the economic value of deposits via Montecarlo simulations of the Formula 9. The standard approach requires to generate a number of simulated paths for the risk factors by means of the estimated dynamics, following these steps:

- compute 10,000 paths for the market rate evolution, simulated with the CIR dynamics;

- for each path, compute the corresponding path for the deposit rate and the deposit volume according to estimated regressions (equations 1 and 2);

- compute deposits value at each time steps in the simulation period;

- sum discounted values path by path and average them to obtain the present value of the total amount of deposits.

Figure 4 shows simulated paths for state variables, using the CIR process with the estimated parameters, starting from the first date after the end of the sample period, the average path of deposits' volume and the minimum amount computed at the 99% c.l. With an initial deposits total volume of $834,468$ bln euros and a simulation period of 10 years, the estimated economic value to the bank of holding deposits is $121,030$ bln. We provide empirical results also for the reduced version of the linear behavioural model given in Equation 3. Table 3 reports regression parameters for this model, and simulated paths are plotted in Figure 5. As expected, excluding the time trend, deposits' volume forecast is much more conservative, and the minimum volume at the 99% c.l. rapidly decreases.

	Coefficient	Significance (p-values)
Intercept Intercept	0.19	0.048
Lagged D_{t-1}	0.98	2.34E-159
Market rate's variations Δr_t	-4.1	2.57E-04
Deposit rate's variation Δd_t	6.52	0.04
R^2	0.99	
F statistics	5837	
F significance	1.66E-157	

TABLE 3: Regression results for the reduced version of the linear behavioural equation 3.

	Coefficient	Significance (p-values)
Intercept	0.25	1.09e-167
Lagged λ_{t-1}	0.53	1.20e-158
Gamma market rates $H(r_t)$	-0.09	4.12e-083
Gamma market rates θ_1	18.77	1.13e-077
Gamma market rates k_1	0.001	3.86e-081
Gamma deposit rates $H(d_t)$	0.14	0.0054
Gamma deposit rates θ_2	24.26	1.67e-066
Gamma deposit rates k_2	0.001	3.01e-056
R^2	0.97	
F statistics	4518	
F significance	1.1767E-157	

TABLE 4: Regression results for the non-linear behavioural equation 8.

We now estimate the parameters of the non-linear behavioural model in Equation 8, via a Non-Linear Least Squares algorithm; we still use the same dataset as above, *i.e.* the sample $3/1999 : 4/2012$ of monthly data for non-maturing deposits volumes, 1-month Eonia swap rates and deposit rates. In this case, what we actually model is the evolution of the proportion λ of the depositor's income held in a sight deposit. At an aggregated level, we approximated the total income with the nominal GDP, so that the fraction λ will be referred to this quantity.Since we are working with Italian deposits, we take the Italian GDP data that are published quarterly and we operate a linear interpolation to obtain monthly values.[4] The reconstructed nominal GDP time series, for the estimation period we consider, is shown in Figure 6 on the left. Estimated coefficients and their significance are shown in Table 4. Figure 7 plots the pdf of the strike respectively for the market (E) and the deposit (F) rates. We can see that the cumulative density functions reach their maximum when the market rate exceeds 3.55% and deposit rate exceeds 4.25%. These should be considered the levels for market interest rates and deposit rates when most of customers consider re-allocating the fraction of income held in deposits on other investments. The regression has an R^2 value lower than the linear model tested before: this is also confirmed by the plot of the actual vs fitted deposits' volumes in Figure 6 on the right. As already done for the linear model, we can compute the economic value of deposits with a Montecarlo simulation. Figure 8 shows simulated paths deposits' volumes and the term structure of expected and minimum volumes. With a simulation period of 10 years and an initial volume of $834,467$ bln Eur, the estimated deposits' economic value is $88,614$, so the non-linear model is more conservative than the linear one. Also for the non-linear model, we can run Montecarlo simulations after freezing the time-trend

[4]We are aware this is likely not the most sound way to interpolate GDP data, but we think it is reasonably good for the limited purpose of our analysis.

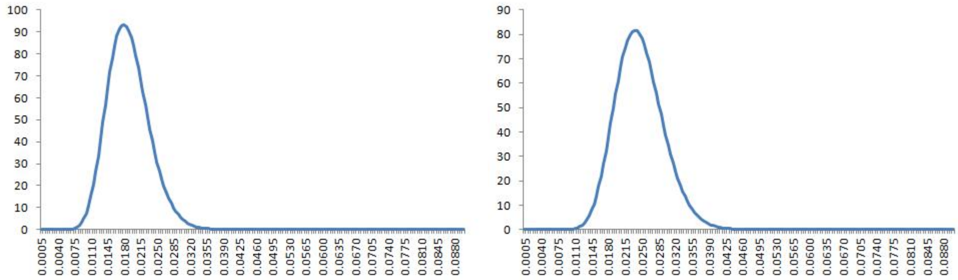

FIGURE 7: Gamma probability density function of the strike level for market interest rates (upper graph) and for deposit rates (lower graph), given the estimated parameters.

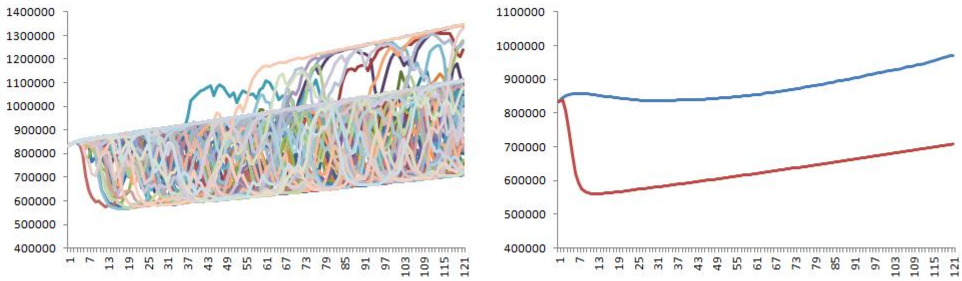

FIGURE 8: Simulated paths (upper graph) and term structure of expected and minimum (99% c.l) future volumes (lower graph) derived with the estimated non-linear model.

(which in this case means keeping the GDP constant to the initial level) and the deposit rate. In this way we isolate the effect produced by the market interest rates on the deposits' volume. The results for the case when only the time-trend is frozen, are shown in Figure 9. It is worth noticing that without time-trend, the fraction of income held in deposits rapidly reach the minimum and then, given the autoregressive nature of the model in Equation 8, it keeps constant at this level. Figure 10 shows the results when both the time trend and the deposit rate are frozen: qualitatively they are the same as in the case when only the time-trend is frozen.

	Sensitivity	Sensitivity
Years	Linear Model	Non-linear Model
1	730	524
2	770	683
3	810	663
4	860	657
5	910	659
6	960	664
7	1000	678
8	1050	689
9	1090	703
10	1100	723

TABLE 5: Sensitivities to 1Y1Y forward Eonia swap rates for 10 bps up to 10 years, for the linear and non-linear model.

A comparison between the linear and non-linear model, as far as the expected and minimum level of deposits' volume are concerned, is shown in Figure 11. It is quite clear that the non-linear model seems to be much more conservative in terms of expected and minimum level of volumes. We present a comparison also of the market rate sensitivities of the economic value of the deposits obtained by the linear and non-linear model. In Table 5 sensitivities to the 1-year forward (risk-free) Eonia rates, fixed every year up to 10 years, are shown. Sensitivities are referred to a bump of the relevant forward rate of 10 basis points. The linear model has bigger sensitivities due to the higher volumes, and hence higher economic value, expected in the future.

Inclusion of Bank-Runs

It can be interesting to include the possibility of a bank-run in the future, due to a lack of confidence of the depositors in the creditworthiness and the accountability of the bank. If this occurs, it is reasonable to expect a sharp and sudden decline in the deposits' volumes. To take into account a bank-run, one needs to consider some variable that is linked to the bank's credit robustness (or the lack of it). One possible solution could be the credit spread of the bank, on short or long-term debt: it can be either extracted from market quotes of the bonds issued by the bank, of from bank's CDS quotes. As for the model, for the very nature of the bank-run, the non-linear behavioural model is more suitable

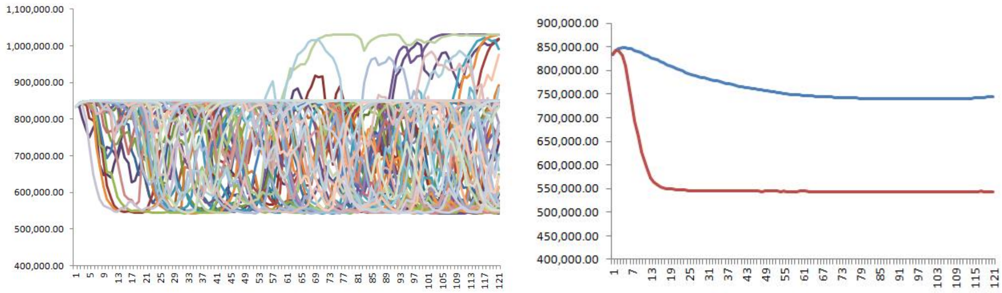

FIGURE 9: Simulated paths (upper graph) and term structure of expected and minimum (99% c.l) future volumes (lower graph) derived with the estimated non-linear model, when the time-trend is frozen.

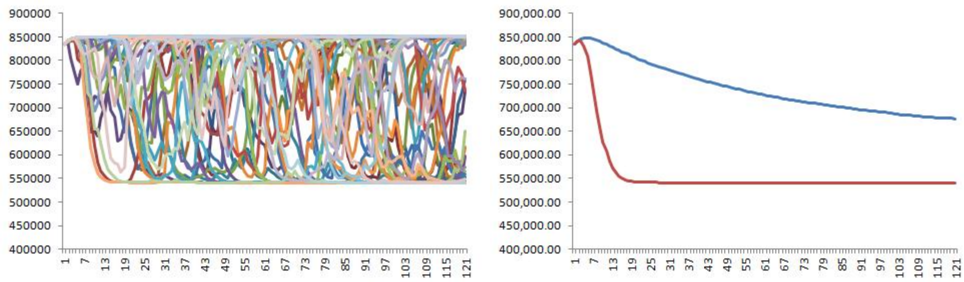

FIGURE 10: Simulated paths (upper graph) and term structure of expected and minimum (99% c.l) future volumes (lower graph) derived with the estimated non-linear model, when the time-trend and the deposit rate are frozen.

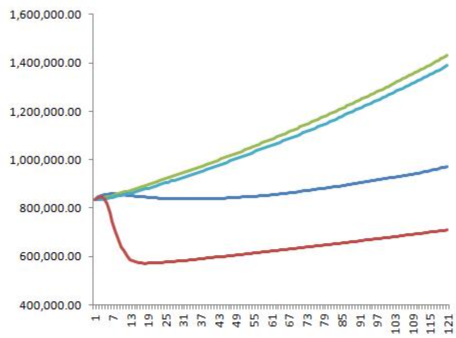

FIGURE 11: Term structure of expected and minimum (99% c.l) future volumes (lower graph) derived with the linear (equation 2) and the non-linear (equation 8) model.

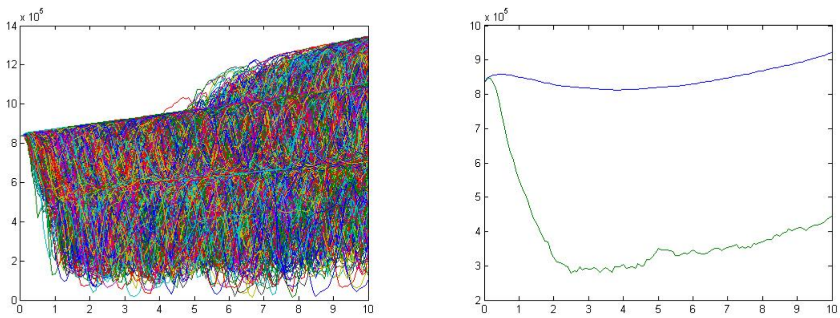

FIGURE 12: Simulated paths (upper graph) and term structure of expected and minimum (99% c.l) future volumes in the case of bank-run inclusion.

to accommodate for it. In fact, it is possible to add an additional behavioural function, related to the bank's credit spread, which will likely be densely concentrated around a high level (denoting an idiosyncratic critical condition). The inclusion of the bank-run is operated by extending formula (8) as follows:

$$\lambda_t = \alpha + \beta\lambda_t + \gamma H(r_t; k_1, \theta_1) + \delta H(i_t, k_2, \theta_2) + \eta H(s_t^B; k_3, \theta_3) \quad (11)$$

The new behavioural function $H(s_t^B; k_3, \theta_3)$ is still a Gamma function taking as an input the bank's spread s^B. It is quite difficult to estimate the parameters of this function, since it is quite unlikely that the bank experienced many bank-runs. One can resort to bank-runs occurred to comparable banks, but also in this case not to many events can be observed for a robust estimation of the parameters. Nonetheless, the bank can include the bank-run on a judgmental basis, by assigning given values to the behavioural function according to its hypothesis of stressed scenarios.

EXAMPLE 3

We extend the non-linear model we estimated in Example to include the possibility of a bank-run. To compute the term structure of expected and minimum volume of deposits, we use Equation (11), with parameters set as shown in Table 4. The parameters of the additional behavioural function are set as follows:

$$\eta = 0.2, \quad k_3 = 32, \quad \theta_3 = 0.002$$

Given the parameters of the Gamma function k_3 and θ_3, when the credit spread of the bank reaches a level above 800 bps, then a drop of 20% in the level of the deposits is experience in each period (we recall we use monthly steps in our examples). To model the credit spread and simulate its evolution in the future, we assume that the default intensity of the bank is given by a CIR process, with parameters:

$$\lambda_0 = -0.2, \quad \kappa = 0.5, \quad \theta = 5\%, \quad \sigma = 12\%$$

Besides we assume a $=$ 60% upon bank's default. We assume that the spread entering in the behavioural function is the 1-month one, for short-term debt. Figure 12 shows the simulated paths and the term structure of the expected and minimum volume of deposits: when compared with Figure 8 it is evident the lower levels projected by the model.

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References

[1] **Jarrow, R.A. and D.R. van Deventer.** *The arbitrage-free valuation and hedging of demand deposits and credit card loans.* Journal of Banking and Finance 22:249-272, 1998.

[2] **Frauendorfer, K. and M. Schürle.** *Dynamic modelling and optimization of non-maturing accounts.* Working Paper. University of St.Gallen, 2006.

[3] **Elkenbracht, M. and B. Nauta.** *Managing interest rate risk for non-maturing deposits* Risk 19(11):82-87, 2006.

[4] **Blöchlinger A.** *Management framework for non-maturing account: from the marketing to the hedging strategy* 2011.

[5] **Janosi, T., R.A. Jarrow and F. Zullo.** *An empirical analysis of the Jarrow - van Deventer model for valuing non-matury demand deposits.* The Journal of Derivatives, 7:8-31, 1999.

[6] **Nyström K.** *On deposit volumes and the valuation of non-maturing liabilities.* Journal of Economic Dynamics & Control, 32:709-756, 2008.

[7] **Boekel P. et al.** *Replicating portfolios.* Milliman Research Report, 2009.

[8] **Matz, L. and P. Neu.** *Liquidity risk measurement and management.* Wiley Finance, 2006.

[9] **Kalkbrener P. and J. Willing.** *Risk management of non-maturing liabilities.* Journal of Banking and Finance 28(7):1547-1568, 2004.