Incremental Valuation of Derivative Contracts

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Incremental Valuation

- For an evaluator, which is a hedger/replicator, define:
  - **Price** of a (derivative) contract: the terms that both parties agree upon when closing the deal.
  - **Value** of a (derivative) contract: the present value of the costs paid to replicate the intermediate and final pay-off until the expiry:
- Only the **value** of a (derivative) contract can be **incremental**; the concept of *incremental pricing* is meaningless.
- Incremental valuation is based on the concept that:

$$A = \neg A$$

- A contract A is A only if it is considered in its relationships with everything else that is *not* the contract A.
- The valuation of a contract A without considering the system it is included within, it an abstract concept.

Incremental Valuation

Incremental with respect to what?

- In theory:
  - the economic net wealth of the *evaluator*, *i.e.*: the bank;
  - the economic net wealth of the bank is its value to the shareholders, *i.e.*: the last claimant on the residual value of the assets.
- In practice:
  - the evaluator can be a desk or an area of the bank (*e.g.*: the dealing room);
  - this is more manageable under a practical point of view, but it may make no sense under an economic point of view.
- We will focus on the theoretical bank value point of view. Results are based on the work by Castagna [1].
- *Nota Bene*: if the bank's management maximises the value to the shareholders, they are also acting in the best interest of all other senior claimants (i.e.: no conflict of interests between shareholders and other claimants).

Investing in One Asset

- We are in an economy with interest rates set constant at zero level.
- An asset $A_1(t)$ has an initial price X_1 in t, terminal pay-off $A_1(T) = X_1 \times (1 + s_1)$, in T. We will assume that $T - t = 1$.
- The asset's issuer can default with probability $\mathbf{PD}_1$ between t and T: the asset's buyer receives a stochastic recovery $\mathbf{Rec}_1$ (a % of face value X_1). $\mathbf{Rec}_1$ takes values $\mathbf{Rec}_{1_j}$, for $j = 1, ..., J$, occurring with probability p_{1_j}.

$$A_1(t) = \mathbf{E}[X_1 \times (1 + s_1)] = X_1 \times (1 + s_1)(1 - \mathbf{PD}_1) + X_1 \overline{\mathbf{Rec}}_1 \mathbf{PD}_1 = X_1 \tag{1}$$

- $\overline{\mathbf{Rec}}_1 = \sum_{j=1}^{J} \mathbf{Rec}_{1_j} p_{1_j}$ is the expected recovery in the event of the issuer's bankruptcy.

Example

In $t = 0$ the asset price is $A_1(0) = 100$. The issuer A_1 can default with probability $\mathbf{PD} = 5\%$. Upon default, the recovery rate is stochastic: possible outcomes and the associated probabilities are in table. The expected loss given default is $\overline{\mathbf{Lgd}}_1 = 1 - \overline{\mathbf{Rec}}_1 = 60\%$

	$\mathbf{Rec}_{1_j}$	p_j
	75%	20%
	35%	70%
	5%	10%
Exp. Recovery $\overline{\mathbf{Rec}}_1$	40%	

Investing in One Asset

- An investor (with zero leverage) buys the asset A_1. In perfect markets, the spread s_1 they require from the issuer of the asset A_1 is simply the fair credit spread cs_1 remunerating the credit risk:

$$s_1 = cs_1 = \frac{\overline{\mathbf{Lgd}}_1 \mathbf{PD}_1}{1 - \mathbf{PD}_1} \qquad (2)$$

where $\overline{\mathbf{Lgd}}_1 = (1 - \overline{\mathbf{Rec}}_1)$ is the loss given default rate (the complementary to 1 of the recovery rate). The spread s_1 is set at the level that makes the terminal expected value of A_1 equal to the present value, X_1.

Example

The fair (credit) spread $s_1 = cs_1$ requested by a non-leveraged investor. This is given by equation (2):

$$s_1 = cs_1 = \frac{60\% \times 5\%}{1 - 5\%} = 3.158\%$$

so that at the expiry the asset has a terminal value of $A_1 = 100 \times (1 + 3.158\%)$.

Investing in One Asset

- Assume now that a bank invests in the same asset A_1. The bank issues a bond to buy the asset.
- We denote the value of the debt at time t with $D_1(t)$ and and the amount of equity posted with E, which is invested in a risk-free bank account, $B(t) = E$.
- The amount needed to fund the asset is $D_1(t) = X_1$: this amount is raised by the bank with a bond issuance.

Example

Assume that the bank starts its activity in $t = 0$ with an amount of equity capital $E = 35$, which is deposited in a bank account $B(0) = 35$. The bank wishes to invest in the asset, whose price is $A_1(0) = 100$ and it issues debt $D_1(0) = 100$ to buy it. A sketched bank's balance sheet is the following:

Assets	Liabilities
$B = 35$ $A_1 = 100$	$D_1 = 100$ ――――― $E = 35$

Investing in One Asset

- In a perfect market:

$$D_1(t) = \mathbf{E}\left[X_1 \times (1 + f_1)\right]$$
$$= X_1 \times (1 + f_1)(1 - \mathbf{PD}_1) +$$
$$\sum_{j=1}^{J} \min[X_1 \mathbf{Rec}_{1_j} + E; X_1 \times (1 + f_1)] p_{1_j} \mathbf{PD}_1$$
$$= X_1$$

- The funding spread is:

$$f_1 = \frac{\overline{\mathbf{Lgd}}_1^* \mathbf{PD}_1}{1 - \mathbf{PD}_1}$$

$\overline{\mathbf{Lgd}}_1^* = 1 - \sum_{j=1}^{J} \min[X_1 \mathbf{Rec}_{1_j} + E; X_1 \times (1 + f_1)] p_{1_j} / X_1$.

Example

The bank is a leveraged investor, since it issues an amount of debt sufficient to buy the asset. Assuming we are in a market where perfect information is available to all participants, then the creditors of the bank know that it will buy the asset A_1 and consequently they set a credit spread on the debt D_1, which is a funding spread for the bank, so that we get:

$$f_1 = 1.406\%$$

Investing in One Asset

- The fair mark-up spread ms_1 the bank has to charge on asset A_1, given the limited liability of the shareholders, is obtained by the equation:

$$\mathbf{VB}(t) = \mathbf{E}\left[X_1 \times (1 + ms_1) + E - X_1 \times (1 + f_1)\right] =$$

$$[(ms_1 - f_1)X_1 + E](1 - \mathbf{PD}_1) + \sum_{j=1}^{J} \max[X_1 \mathbf{Rec}_{1_j} + E - X_1 \times (1 + f_1); 0] p_{1_j} \mathbf{PD}_1$$

$$= E \tag{3}$$

The net value of the bank $\mathbf{VB}(t)$ at time t is equal to the expected value of the future value in T of

1. *bank's total assets* $= X_1 + ms_1 X_1$ (the margin) $+ E$ (the equity amount in the risk-free account);
2. minus *bank's total liabilities* $= X_1 + f_1 X_1$ (the funding costs).

The bank's shareholders invested the initial amount E, so $\mathbf{VB}(t)$ must equal E.

Investing in One Asset

- Indicating the *average recovery* on the bank value with

$$\overline{R}_1 = \sum_{j=1}^{J} \max[X_1 \mathbf{Rec}_{1_j} + E - X_1 \times (1 + f_1); 0] p_{1_j},$$

the mark-up spread, from (3), is:

$$ms_1 = \frac{\frac{E - \overline{R}_1}{X_1} \mathbf{PD}_1}{1 - \mathbf{PD}_1} + f_1 = cs_1^* + f_1 \quad (4)$$

This is the sum two components:

  - the "adjusted" credit spread $cs_1^* < cs_1$ on the asset A_1 (since the loss given default $(E - \overline{R}_1)/X_1$ is lower than $\mathbf{Lgd}_1$; the smaller loss given default is produced by the leveraged investment in the asset A_1, and by the limited liability up to E; a share of the **Lgd** is taken by the debt holders);
  - the funding spread f_1 paid by the bank on its debt.

Investing in One Asset

- By some manipulations, it is easy to check that in perfect markets where the credit spreads set by investors are fair and given in (2), we have:

$$ms_1 = \frac{\frac{E-\overline{R}_1}{X_1}\mathbf{PD}_1 + \overline{\mathbf{Lgd}}_1^*\mathbf{PD}_1}{1-\mathbf{PD}_1} = \frac{\overline{\mathbf{Lgd}}_1\mathbf{PD}_1}{1-\mathbf{PD}_1} = s_1 \quad (5)$$

- The mark-up spread is just the credit spread of the asset A_1 required by a for a non-leveraged investor:

Proposition

If the bank holds only one asset, the leverage is immaterial in its internal pricing by the bank. Differently stated, the bank can price the asset as it were an non-leveraged investor, and the assets' price would depend only on its expected future pay-off.

- This result is definitely not new: it is the same as the well known works by Modigliani&Miller (M&M) [3] and Merton [2].

Investing in One Asset

Example

We can now compute the fair margin ms_1 that the bank should charge on asset A_1, by means of formula (4). First, we compute the different R_{1_j}s, shown in the table below. By these quantities we can compute also the bank's default probability: this is shown as well.

R_{1_j}	PD_B
1.719	0.000%
-	3.500%
-	0.500%
1.719	4.000%

The "adjusted" credit spread cs_1^*,

$$cs_1^* = \frac{\frac{35-1.79}{100}5\%}{1-5\%} = 1.752\%$$

which plugged in (4)

$$ms_1 = cs_1^* + f_1 = 1.752\% + 1.406\% = 3.158\% = s_1$$

thus confirming (5).

Investing in Two Assets

We move on to a multi-period setting:

- The bank, after the investment in A_1, decides to invest in a new asset A_2, whose initial price is X_2 and terminal pay-off $A_2(T) = X_2 \times (1 + s_2)$.
- We assume that the expiry of the asset A_2 is in T (same as asset A_1) and that the investment occurs in t^+, just an instant after the initial time t.
- We set $t^+ = t$ in what follows, even though they are two distinct instants.
- For asset A_2 there is a probability $\mathbf{PD}_2$ that the asset's issuer defaults: the buyer of the asset receives a stochastic recovery $\mathbf{Rec}_2$.
- $\mathbf{Rec}_2$ can take values $\mathbf{Rec}_{2_l}$, for $l = 1, ..., L$, and each possible value can occur with probability p_{2_l}. We assume that the defaults of the issuers of A_1 and A_2 are uncorrelated.
- The bank buys the asset $A_2(t) = X_2$ by issuing new debt: the total debt is $D(t) = D_1(t) + D_2(t) = X_1 + X_2 = X$, *i.e.*: the leverage increases as well.

Investing in Two Assets

- Let f_2 be the funding spread paid on debt $D_2(t)$.
- The funding spread requested by bank's bond holders on the new debt $D_2(t)$ is derived in a way similar to equation (9):

$$
\begin{aligned}
D_2(t) &= \mathbf{E}\left[X_2(1+f_2)\right] \\
&= X_2(1+f_2)(1-\mathbf{PD}_1)(1-\mathbf{PD}_2) \\
&+ \sum_{j=1}^{J} \min\left[\frac{X_1\mathbf{Rec}_{1_j} + X_2(1+s_2) + E}{X}; (1+f_2)\right] X_2 p_{1_j}\mathbf{PD}_1(1-\mathbf{PD}_2) \\
&+ \sum_{l=1}^{L} \min\left[\frac{X_1(1+s_1) + X_2\mathbf{Rec}_{2_l} + E}{X}; (1+f_2)\right] X_2 p_{2_l}\mathbf{PD}_2(1-\mathbf{PD}_1) \\
&+ \sum_{j=1}^{J}\sum_{l=1}^{L} \min\left[\frac{X_1\mathbf{Rec}_{1_j} + X_2\mathbf{Rec}_{2_l} + E}{X}; (1+f_2)\right] X_2 p_{1_j} p_{2_l}\mathbf{PD}_2\mathbf{PD}_1 \\
&= X_2
\end{aligned}
\tag{6}
$$

- The funding spread f_2 can be found by solving equation (6):

Investing in Two Assets

- The mark-up margin, set by the bank on the second asset, is such that the expected net value of the bank is still the amount of equity posted by shareholders:

$$\begin{aligned} \mathbf{VB}(t) =& \mathbf{E}\left[X_1(ms_1 - f_1) + X_2(ms_2 - f_2) + E\right] \\ &= \left[X_1(ms_1 - f_1) + X_2(ms_2 - f_2) + E\right](1 - \mathbf{PD}_1)(1 - \mathbf{PD}_2) \\ &+ \overline{R}_1^* \mathbf{PD}_1(1 - \mathbf{PD}_2) + \overline{R}_2^* \mathbf{PD}_2(1 - \mathbf{PD}_1) \\ &+ \overline{R}_{1,2}^* \mathbf{PD}_2 \mathbf{PD}_1 = E \end{aligned} \tag{7}$$

$$\overline{R}_1^* = \sum_{j=1}^{J} \max\left[X_1(\mathbf{Rec}_{1_j} - (1 + f_1)) + X_2(ms_2 - f_2) + E; 0\right] p_{1_j},$$

$$\overline{R}_2^* = \sum_{l=1}^{L} \max\left[X_1(ms_1 - f_1) + X_2(\mathbf{Rec}_{2_l} - (1 + f_2)) + E; 0\right] p_{2_l}$$

$$\overline{R}_{1,2}^* = \sum_{j=1}^{J} \sum_{l=1}^{L} \max\left[X_1(\mathbf{Rec}_{1_j} - (1 + f_1)) + X_2(\mathbf{Rec}_{2_l} - (1 + f_2)) + E; 0\right] p_{1_j} p_{2_l}.$$

Cpty Default Does not Trigger Banks' Default

- Consider the case when the default of the asset A_2 does not imply the default of the bank. This may happen because the quantity X_2 is small compared to the entire balance sheet.
- In the event of bankruptcy of A_2, the bank is able to cover losses and to repay all its creditors without depleting its equity capital E. The default of the asset A_1 still causes the default of the bank as before.
- If X_2 is much smaller than the quantity X_1 of asset A_1 already included in the assets of the bank's balance sheets, then we have the following approximations:

$$\overline{R}_1^* \approx \overline{R}_{1,2}^* \approx \overline{R}_1$$

and

$$\overline{R}_2^* \approx X_1(ms_1 - f_1) + X_2(\overline{\mathbf{Rec}}_2 - (1 + f_2)) + E$$

- Replacing in (7), we get

$$ms_2 = \frac{\overline{\mathbf{Lgd}}_2 \mathbf{PD}_2 + f_2}{1 - \mathbf{PD}_2} = cs_2 + \frac{f_2}{1 - \mathbf{PD}_2} \quad (8)$$

Cpty Default Does not Trigger Banks' Default

Proposition

When pricing an asset that represents a small percentage of the bank's total assets and whose default does not affect the bank's default, the correct (approximated) and theoretically consistent mark-up margin to apply includes the issuer's credit spread, fair to a non-leveraged investor, plus the bank's funding spread conditioned to the issuer's survival probability.

- An example is given by an asset representing a small percentage of the total bank's assets: when its issuer defaults, this bankruptcy would not affect the survival of the bank.
- Modigliani& Miller and Merton results are valid: the leverage does not matter at an aggregate level (*i.e.*: the expected value of the bank is always the current value of equity E), but it **does** matter to evaluate contracts incrementally added in the bank's balance sheet.

Cpty Default Triggers Banks' Default

- Assume that the default of the asset A_2 implies the default of the bank: this is the case when the asset A_2 represents, in percentage terms, a great share of the bank's total assets.
- The fair margin spread is derived by solving the bank's value equation:

$$\begin{aligned}\mathbf{VB}(t) &= \mathbf{E}\left[X_1(ms_1 - f_1) + X_2(ms_2 - f_2) + E\right] \\ &= \left[X_1(ms_1 - f_1) + X_2(ms_2 - f_2) + E\right](1 - \mathbf{PD}_1)(1 - \mathbf{PD}_2) + \epsilon \\ &= E\end{aligned} \tag{9}$$

where $\epsilon = \overline{R}_1^* \mathbf{PD}_1(1 - \mathbf{PD}_2) + \overline{R}_2^* \mathbf{PD}_2(1 - \mathbf{PD}_1) + \overline{R}_{1,2}^* \mathbf{PD}_1 \mathbf{PD}_2$. We solve for ms_2 and we get:

$$\begin{aligned}ms_2 &= \frac{\frac{E - (1-\mathbf{PD}_1)(1-\mathbf{PD}_2)[E + X_1(ms_1 - f_1)] - \epsilon}{X_2}}{(1 - \mathbf{PD}_1)(1 - \mathbf{PD}_2)} + f_2 \\ &= cs_2^* + f_2\end{aligned} \tag{10}$$

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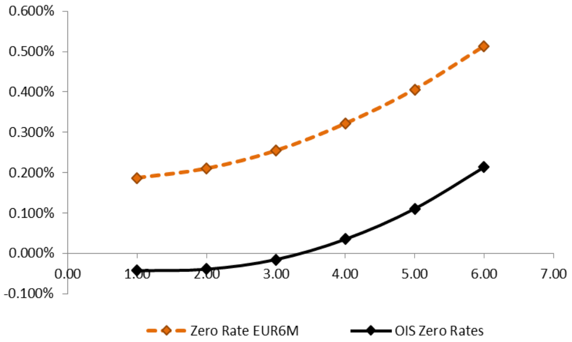
Set-Up

- Evaluation horizon T is equal to, or greater than, the expiry of the longest maturing contract.
- The bank trades with $j = 1, ..., J$ counterparties: so there are J netting sets.
- Each netting set j contains $i_j = 1, ..., I_j$ contracts, V_{i_j}. The value of the netting set is $V_j = \sum_{i=1}^{I} V_{i_j}$.
- Netting sets can be collateralised with collateral C_j. The value V_{i_j} includes collateral (=NPV - Collateral)
- Assets' and liabilities' cash-flows $\mathbf{cf}(t_m)$ are deposited/withdrawn in/from a bank account B and earns the risk-free rate r.
- Bank account can never be below 0, so when cumulative cash-flows of existing contracts imply a negative balance, short-term debt (liabilities) $SL(t) = \left| \min \left[B(t), 0 \right] \right|$ is issued.
- At time 0 the bank has some long-term debt outstanding LL, expiring in $T_L \leq T$. We assume it is rolled over for a period equal to T_L, or up to T if the roll-over outlive the evaluation horizon.

Market Data

- Market Curves

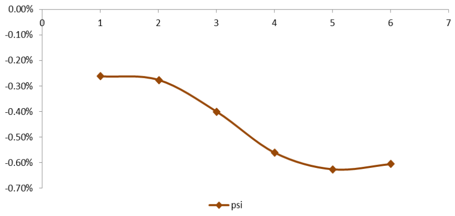


Interest Rate Modelling

- CIR++ model with parameters for OIS rates:

r_0	0.01%
κ	0.045
θ	5.56%
σ	4.45%

- Time dependent parameter $\psi(t)$



- The 6M Libor-OIS basis is kept constant.

Bank Value at time 0

- We do not assume that the bank starts in 0, so its value is not simply the initial equity E: it is the net of assets and liabilities.
- The bank's default can be triggered not just by the counterparties' default but also by the economic results of the exiting contracts.
- Let the bank's default time τ^B be:

$$\tau_B = \inf\{0 \leq t \leq T : \mathbf{VB}(t) < 0\}$$

- The value of the bank at time 0 is:

$$\mathbf{VB}(0) = \mathbf{E}[D(0,T)\mathbf{VB}(T)\mathbf{1}_{\{\tau_B > T\}}]$$

where $D(0,s) = e^{-\int_0^s r_s ds}$ is the discount factor.

- Numerical simulations are needed.

Balance Sheet Modelling

- The present value in 0 of the bank account (initial amount $B(0) \geq 0$) up to t is:

$$\mathcal{B}(0,t) = B(0) + \sum_{m=1}^{M} \mathbf{cf}(t_m) \int_0^t D(0,t_m)\delta(t_m - s)ds + \int_0^t D(0,s)B(s)r_s ds$$

  where $\delta(s)$ is a Dirac function centered in s.

- The present value of the short term debt in 0 up to t:

$$\mathcal{S}(0,t) = SL(t)D(0,t) + \int_0^t D(0,s)SL(s)[r_s + s_s^B]ds$$

- Long term debt (bond with annual coupon) outstanding at time t is evaluated considering that it is rolled over up to T and that the coupon paid can change on each roll-over date. The present value in 0 is:

$$\mathcal{L}(t,T) = \sum_{n=1}^{N} \kappa(t_n)LL \int_t^T D(0,t_m)\delta(t_m - s)ds + D(0,T)LL$$

  where $\kappa(t_n)$ is the coupon paid on the notional L; $N =$ numbers of coupons to pay between t and $t_N = T$.

Balance Sheet Modelling

- To make things more tractable, we assume that the joint default of two counterparties k and h is zero, so that:

$$P(\tau_k = \tau_h) = 0$$

where τ_j is the default time of the counterparty j.

- Define $\mathbf{VB}_j(\tau_j)$ the value of the bank that include also the loss given default of the counterparty j at time τ_j.
- The value of the bank is:

$$\mathbf{VB}(0) = \mathbf{E}\Bigg[\Bigg[\sum_j D(0,T)\mathbf{VB}(T)(1 - \mathbf{1}_{\{\tau_j < T\}}) + \sum_j \max[D(0,\tau_j)\mathbf{VB}_j(\tau_j), 0]\mathbf{1}_{\{\tau_j < T\}}\Bigg]\mathbf{1}_{\{\tau_B > T\}}\Bigg]$$

or

$$\mathbf{VB}(0) = \mathbf{E}\Bigg[\Bigg[D(0,T)\mathbf{VB}(T) - \left[D(0,\tau_j)\mathbf{VB}(\tau_i) - \sum_j \max[D(0,\tau_j)\mathbf{VB}_j(\tau_j), 0]\right]\mathbf{1}_{\{\tau_j < T\}}\Bigg]\mathbf{1}_{\{\tau_B > T\}}\Bigg]$$

Balance Sheet Modelling

- Since **cf** are already computed in the contracts' values, at any t we need only the starting value and the interest accrued up to T on the bank account:

$$\mathcal{B}^*(t,T) = \mathcal{B}(0,t) + \int_t^T D(0,s)B(s)r_s ds$$

- We introduce the costs related to the collateral as:

$$\mathcal{C}_j(0,t) = \int_0^t D(0,s)C_j(s)(r_s - c_s)ds$$

- More explicitly, the value of the bank is:

$$\begin{aligned}\mathbf{VB}(0) =& \mathbf{E}\Bigg[\Bigg[\sum_j D(0,T)V_j(0,T) + \mathcal{C}_j(0,T) + \mathcal{B}^*(0,T) - [\mathcal{L}(0,T) + \mathcal{S}(0,T)] - \\ & \sum_j \mathbf{1}_{\{\tau_j<T\}}\big[D(0,\tau_j)V_j(\tau_j,T) + \mathcal{C}_j(0,\tau_j) + \mathcal{B}^*(\tau_j,T) - [\mathcal{L}(\tau_j,T) + \mathcal{S}(0,\tau_j)] - \\ & \max[\sum_{k\neq j} D(0,\tau_j)V_k(\tau_j) + \mathcal{C}_k(0,\tau_j) + \mathcal{B}^*(\tau_j,T) \\ & + D(0,\tau_j)(\mathbf{Rec}\, V_j^+(\tau_j,T) + V_j^-(\tau_j,T)) - [\mathcal{L}(\tau_j,T) + \mathcal{S}(0,\tau_j)], 0]\big]\Bigg]\mathbf{1}_{\{\tau_B>T\}}\Bigg]\end{aligned}$$

Note that all (cash) collateral is in the bank account.

Balance Sheet Modelling

- Define the *xVA*s adjusted for the shareholders' limited liability as:

$$\mathbf{CVA}_j^{LL}(0,T) = \mathbf{E}\Bigg[\mathbf{1}_{\{\tau_j<T\}}\Bigg(D(0,\tau_j)V_j(\tau_j,T) + \mathcal{C}_j(0,\tau_j) + \mathcal{B}^*(\tau_j,T)$$
$$- [\mathcal{L}(\tau_j,T) + \mathcal{S}(0,\tau_j)] - \max\Big[\sum_{k\neq j} D(0,\tau_j)V_k(\tau_j,T) + \mathcal{C}_k(0,\tau_j) + \mathcal{B}^*(\tau_j,T)$$
$$+ D(0,\tau_j)(\mathbf{Rec}\,V_j^+(\tau_j,T) + V_j^-(\tau_j,T)) - [\mathcal{L}(\tau_j,T) + \mathcal{S}(0,\tau_j)], 0\Big]\Bigg)\mathbf{1}_{\{\tau_B>T\}}\Bigg]$$

$$\mathbf{LVA}_j^{LL}(0,T) = \mathbf{E}\left[\left(\int_0^t D(0,s)C_j(s)(r_s - c_s)ds\right)\mathbf{1}_{\{\tau_B>T\}}\right]$$
$$\mathbf{FVA}^{LL}(0,T) = \mathbf{E}\left[\left(\int_0^t D(0,s)SL(s)s_s^B ds\right)\mathbf{1}_{\{\tau_B>T\}}\right]$$

- and setting

$$\mathcal{S}^*(0,t) = SL(t)D(0,t) + \int_0^t D(0,s)SL(s)r_s ds$$

Balance Sheet Modelling

- We can then write the value of the bank as:

$$\mathbf{VB}(0) = \mathbf{E}\left[\left(\sum_j D(0,T)V_j(T) + \mathcal{B}^*(0,T) - [\mathcal{L}(0,T) + \mathcal{S}^*(0,T)]\right)\mathbf{1}_{\{\tau_B > T\}}\right]$$
$$- \mathbf{FVA}^{LL}(0,T) + \sum_j \left[\mathbf{LVA}_j^{LL}(0,T) - \mathbf{CVA}_j^{LL}(T)\right]$$

- A new contract $(I+1)_J$, of the $J-$th netting set, has to be valued incrementally w.r.t the variations of **VB**.

Incremental Pricing

- Assume at time $t > 0$ a new contract V_{I+1_J} is included in the bank's balance sheet. Let $\mathbf{VB}^{+}(t)$ be the value of the bank that includes the new contract.
- The variation of the bank's value $\Delta\mathbf{VB}(t) = \mathbf{VB}^{+}(t) - \mathbf{VB}(t)$ can be decomposed as:

$$\Delta\mathbf{VB}(t) = \mathbf{E}\left[\Delta V_J(t)\mathbf{1}_{\{\tau_B > T\}}\right] - \Delta\mathbf{FVA}^{LL}(0,T) + \sum_j \Delta\mathbf{LVA}_j^{LL}(0,T) - \Delta\mathbf{CVA}_j^{LL}(0,T)$$

- If the contract is fairly priced, then $\Delta\mathbf{VB}(t) = 0$.
- One way to obtain this is to set $B(t) = -\Delta\mathbf{VB}(t)$.

Example: Starting Balance Sheet

- Available cash: -30,568.49
- Swap in the bank's book:

	NOTIONAL	EXP DATE	FIX RATE	FIX FREQ	FLOAT RATE	FLOAT FREQ	COLLATERAL
Swap 1	1,000,000	12/11/2016	2	6m	EURIBOR 6M	6m	C
Swap 2	- 1,100,000	05/06/2017	2.75	6m	EURIBOR 6M	6m	U

- The risk-free NPVs of the 2 swaps are:

	Risk-free NPV	Coll Rate Sprd	PD	CVA	DVA	BCVA	LVA
Swap 1	-35,582.684	-0.10%	0.50%	0.00	0.00	0.00	-43.62
Swap 2	68,444.778	0.00%	1.00%	326.83	0.00	326.83	0.00

- Long-term funding (NPV at risk-free rate): 1,153.10
- Short-term funding needed to cover negative cash: 30,568.49
- The funding spread on the s.t. funding is 1% (simplifying assumption).

Example: Initial Value of the Bank

- The Value of the bank **VB**(0), computed as explained above, is: 36,166.61.
- The marked-to-market (from the bank perspective) the balance-sheet is:

Assets		Liabilities	
Cash+coll	0.00	LT Funding	1,153.10
	-	ST Funding	30,568.49
Swap Book NPV	68,444.78	$\mathbf{CVA}^{LL}$ swap 2	278.84
		$\mathbf{FVA}^{LL}$	234.13
		$\mathbf{LVA}^{LL}$	43.62
		——————————-	
		Value of Equity (**VB**)	36,166.61

- Note that $\mathbf{CVA}_2^{LL} < \mathbf{CVA}_2$, but this is not due to the netting with **DVA**, which is 0.

Example: New Swaps

- The bank trades with a client a new swap, struck at par rate.
- The swap is hedged with an equal and opposite swap, traded with another bank.
- We fist consider the case a small amount is traded:

	NOTIONAL	EXP DATE	FIX RATE	FIX FREQ	FLOAT RATE	FLOAT FREQ	COLLATERAL
Par Swap	100,000	05/11/2019	0.401765	6m	EURIBOR 6M	6m	U
Hedge Swap	-100,000	05/11/2019	0.401765	6m	EURIBOR 6M	6m	C

- The stand-alone pricing of the two swaps:

	Risk-free NPV	Coll Rate Sprd	PD	CVA	DVA	BCVA	LVA
Par Swap	0.00	0.00	1.00%	17.43	2.34	15.09	0.00
Hedge Swap	0.00	-0.10%	0.75%	0.00	0.00	0.00	-2.03

Example: Incremental Evaluation of the Swap

- The updated bank value is $\mathbf{VB}(0) = 36,156.14$.
- The marked-to-market (from the bank perspective) the balance-sheet is:

Assets		Liabilities	
Cash+coll	0.00	LT Funding	1,153.10
	-	ST Funding	30,568.49
Swap Book NPV	68,444.78	$\mathbf{CVA}^{LL}$ swap 2	278.84
		$\mathbf{CVA}^{LL}$ Par Swap	17.35
		$\mathbf{FVA}^{LL}$	229.25
		$\mathbf{LVA}^{LL}$	41.61
		Value of Equity (**VB**)	36,156.14

Example: Incremental Evaluation of the Swap

- The incremental value of the par-swap is:

	Final	Initial	Δ
V_J	0.00	0.00	-
CVA^{LL} swap 2	278.84	278.84	0.01
CVA^{LL} par swap	17.35	0.00	17.35
FVA^{LL}	229.25	234.13	- 4.88
LVA^{LL}	41.61	43.62	- 2.01
Total Net Cost			10.47

- So $\Delta\mathbf{VB}(0) = -10.47$ has to be charged to the counterparty (as a spread or as upfront cash payment).
- It has to be noted that the **CVA**^{LL} = **CVA** stand alone. No **DVA**.
- The **FVA**^{LL} and **LVA**^{ll} have a positive effect in this case. The **LVA**^{LL} is equal to the one in the stand-alone evaluation.

Example: New Swaps

- Assume now the par swap is traded in $1,000,000$. An equal, opposite amount is traded as a hedge
- The stand-alone pricing of the two swaps:

	Risk-free NPV	Coll Rate Sprd	PD	CVA	DVA	BCVA	LVA
Par Swap	0.00	0.00	1.00%	174.31	23.44	150.88	0.00
Hedge Swap	0.00	-0.10%	0.75%	0.00	0.00	0.00	-20.25

Example: Incremental Evaluation of the Swap

- The updated bank value is $\mathbf{VB}(0) = 36,053.87$.
- The marked-to-market (from the bank perspective) the balance-sheet is:

Assets		**Liabilities**	
Cash+coll	0.00	LT Funding	1,153.10
	-	ST Funding	30,568.49
Swap Book NPV	68,444.78	$\mathbf{CVA}^{LL}$ swap 2	278.91
		$\mathbf{CVA}^{LL}$ Par Swap	169.44
		$\mathbf{FVA}^{LL}$	197.42
		$\mathbf{LVA}^{LL}$	23.51
		Value of Equity (**VB**)	36,053.87

Example: Incremental Evaluation of the Swap

- The incremental value of the par-swap is:

	Final	Initial	Δ
V_J	0.00	0.00	-
$\mathbf{CVA}^{LL}$ swap 2	278.91	278.84	0.07
$\mathbf{CVA}^{LL}$ par swap	169.44	0.00	169.44
$\mathbf{FVA}^{LL}$	197.42	234.13	- 36.71
$\mathbf{LVA}^{LL}$	23.51	43.62	- 20.11
Total Net Cost			112.69

- So $\Delta\mathbf{VB}(0) = -112.69$ has to be charged to the counterparty (as a spread or as upfront cash payment).
- It has to be noted that the $\mathbf{CVA}^{LL} < \mathbf{CVA}$ stand-alone, still $\mathbf{CVA}^{LL} > B\mathbf{CVA}$.

Conclusions

- Incremental valuation is the only way to correctly assess the value of a contract to the bank.
- Stand-alone valuation, or incremental w.r.t to netting sets smaller than the bank's balance sheet can be highly misleading.
- Current accounting rules are not considering the incremental pricing: they rely on the B**CVA**. An widely agreed view on **FVA** not reached yet.
- Computation is cumbersome and demanding, but current technology make it possible.

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