Risk and Profitability of Sight Deposits in the Italian Banking Industry

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Abstract

This paper presents a stochastic risk factor approach to gauge the expected profitability and the liquidity and duration risks of sight deposits of the Italian banking industry, using public data available from Banca d’Italia, spanning over a long period of time that includes the Euro Crisis. The approach is applied to both retail and corporate customers, and it considers their different behaviour based on the size of their deposit.

1 Introduction

The modelling of deposits and non-maturing liabilities is a crucial task for the liquidity and asset and liability management of a financial institution. It has become even much more momentous in the current environment after the liquidity crisis that struck the interbank money market in 2008/2009.

Typically, the ALM departments of banks involved in the management of interest rate and liquidity risks face the task of forecasting deposit volumes, so as to design and implement consequent liquidity and reinvestment strategies. Moreover, deposit accounts represent the main source of funding for the bank, primarily for those institutions focused on the retail business, and they heavily contribute to the funding available for lending activity. Of the different funding sources, sight deposits usually

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‡This is a preliminary version. Comments are welcome.
have the lowest costs, so that in a funding mix they help reducing the total cost of 
funding.

A renewed attention to the modelling of non-maturing liabilities has arisen after 
the publication of a consultative document [10] by the Basel Committee, and even-
tually of the new standards [11] on the measurement and the management of the 
Interest Rate Risk of the Banking Book (IRRBB).

The final standards provide for a standardised approach to measure the risks 
related to behavioural optionalities, including non-maturing liabilities and prepay-
ment of mortgages, yet they allow banks to use internal models, if approved by 
competent authorities.

Lately, updating modelling is felt by banks as urgent because of the low, or even 
negative, interest rate environment. Banks are not passing yet the cost of negative 
rates on depositors, fearing a withdrawal of the deposited amounts and an increase 
of the cash held outside the banking system. Nonetheless, this policy is contributing 
to the shrinking of the net interest income (NII).

For these reasons, we think it is useful to analyse by a robust, general, and 
forward-looking approach the liquidity risk, interest rate exposure, and the prof-
itability of this class of liabilities for the banking industry, focussing our attention 
on the public data available from Banca d’Italia for the Italian banking system.

Our approach follows the methodology introduced by Jarrow and van Deventer 
[7]; Castagna and Fede [4], amongst many others, applied a similar approach to 
the publicly available aggregated data for the Italian banking industry, denoting 
it as stochastic factor (SF), since it identifies statistically how the evolution of 
deposit volumes, and of deposit rates, is linked to risk factors. The dynamics of risk 
factors are also modelled so that, by simulating their future evolution up to a given 
predefined time (cut-off horizon), it is possible to derive the implied dynamics of 
the deposits and of the interest rate paid on them, and the risk metrics related to 
liquidity and interest rates.

The analysis in Castagna and Fede [4] will be extended in what follows in two 
directions. First, we disentangle the data by identifying two types of depositors, 
namely: corporate and retail. Second, we classify the data for each type of depositors 
in five categories according to the amount of the single deposit. In this way, we 
will be able to detect how the two types of depositors, in each category, react to 
the changing of the risk factors. Therefore, we will analyse 8 different clusters of 
depositors.

The SF approach we implement is different from the model usually adopted by 
banks in managing sight deposits, as we will see in Section 2, where we will present 
a comparison between the two frameworks.

We will assess the expected profitability of the sight deposit accounts through 
the computation of the Economic Value; additionally, we will calculate the Duration 
of the deposits’ volume, a measure related to interest rate risk; and finally we will 
compute the Weighted Average Life and the Term Structure of Liquidity, at several 
confidence levels, as measures of the liquidity risk useful to assess the stability of 
the deposits’ volume.

2 Overview of the Approach

In modelling the sight deposits, we adopt an approach that hinges on the main 
idea that their volumes depend on stochastic risk factors. The dependencies are
The approach is made of the following building-blocks:

- the deposits’ volume;
- the deposits’ interest rates;
- the market interest rates;
- the creditworthiness of the average Italian bank, for which we use a 5Y CDS (credit default swap) index as a proxy (see below for the details).

The first and the second building-block are the variables we wish to analyse; the remaining two building-blocks that are related to market stochastic risk factors.

In Figure 1 we sketch the entire framework of the approach. The historical data for the risk factors, the deposit rate and volume, are fed to the calibration engine: the models are calibrated in a nested fashion. First, we estimate the model for the deposit interest rate, which depends on the two stochastic risk factors, i.e.: the market risk-free (short-term) interest rate and the CDS index, representing the credit riskiness of the banking industry. Secondly, we estimate the model for deposit volume, which depends on the market interest rate, the deposit interest rate, and the CDS index. Finally, the dynamics of the two stochastic risk factors, i.e.: the market interest rate and the CDS index, are calibrated to their historical time series. The two risk factors do not depend on any other variables.

Once the calibration is performed, the models can be used to simulate future paths of all the variables: also in this case, the paths are generated in a sort of nested fashion. More specifically, the two risk factors are projected by Montecarlo

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1 The approach is not very different, in spirit, from the one adopted in some other studies; for a very short and incomplete list, we mention the works by Jarrow and Van Deventer [7], Kalkbrener and Willing [8], Dewachter, Lyrio and Maes [6], Nystrom [9] and Blochlinger [1]. For a more general overview, see also Castagna and Fede, chapter 9, [4].
simulations: the values in each path are input in the dynamics of the deposit interest rate, so to produce a Montecarlo simulation also for it. Finally, the two risk factors and the deposit interest rate simulated in each path are input into the dynamics of the deposit volume, to obtain a Montecarlo simulation.

Hence, the simulation of a (high) number of paths yields a distribution for the relevant market risk factors and the deposits’ interest rate and volume. This allows to compute useful quantities for the profitability monitoring and risk management, such as the Economic Value, the Term Structure of Liquidity, the Duration and the Weighted Average Life, which will all be described in the following. The approach can be even employed to generate a consistent scenario analysis, where one can assume some levels for the risk factors that, in turn, will provide the evolution of the deposit interest rate and volume, and the corresponding values of all the metrics we have just mentioned.

The approach we follow is rather different from the approach typically followed within banks, which is also implicitly alluded to by the new Basel documents on IRBBB ([10] and [11]), and which for this reason we refer to as the Basel standard approach. We sketch the framework of the approach in Figure 2: the database of historical data is basically the same, even though only the short term market interest rate is considered as a risk factor, and no other variable related to the creditworthiness of the bank is taken into account.

The calibration is operated at two separate levels: first, the deposit interest rate dynamics, which depends only on the market interest rate, is estimated by a linear relationship or by an Error Correction Model. Then, the dynamics for the deposit volume is in exogenous way superimposed (e.g.: a Geometric Brownian Motion), without any dependence on the market and the deposit interest rates: for this dynamics, often after removing the drift, the volatility parameter is historically estimated. Finally, given the volatility of the time series, the deposit volume at the reference date is separated in a volatile and a stable component. The former is kept in a liquidity buffer, whereas the latter is subsequently divided in a non-core component, allocated in a rolling short-term investment, and a core component invested in a portfolio of medium-long term bonds, based on the amortisation profile derived by a stressed profile of the deposit volume’s dynamics, for example projecting in the future the volume at a 2.65 times the volatility, up to a predefined cut-off time. The core and non-core components are determined according to the sensitivity of the deposit interest rate to the market interest rate.

This approach is more simplified than the SF approach, hence it does not allow for a proper evaluation of the profitability of the sight deposits, nor it provides risk metrics, such as the Duration, or sensitivities to market rates. Moreover, within its framework, a scenario analysis can be run on a limited basis since the volume does not depend on any market variable.

Additionally, in the current low (or even negative) market interest rate environment, the Basel standard approach may produce as outcome some insensible results, by construction of the way in which the core and non-core components are derived.

Finally, it is worth noting that the SF approach models in a unified fashion both the liquidity and the interest rate risks: all the dynamics of deposits’ volume and interest rate depend on risk factors, so they are both consistently affected by those. On the contrary, the standard approach in practice relies on two separate modelling, i.e.: the volume and the deposits’ interest rate, and there is no interrelation whatsoever between the two models. This may create some
For all these reasons, we think the SF approach is superior and more powerful to deal with several market environment, including the current one.

2.1 Deposits Volume

As mentioned above, the deposits volume is assumed to be dependent on a set of risk factors. The main principle in identifying the risk factors affecting the deposits’ evolution is the depositors’ liquidity preferences: the lower the market interest rates, the higher the depositors’ propensity to keep cash and high-liquidity assets, such as sight deposits. So a negative relation is expected to be found between the level, or variations, of the volume of deposits and the level, or variations, of interest rates when estimating it with historical data.

The effects of the market interest rates can be counterbalanced by the deposits’ interest rates: the higher they are, the more depositors are encouraged to keep their wealth in deposits, since their yield competes with other types of investments.

Finally, the depositors are expected to withdraw from their deposits as the risk of the bank’s default increases. We proxy this risk with the Credit Default Swap premium quoted in the market. Since we are dealing with data referring to the entire Italian banking industry, we build a CDS index as explained below.

We model the evolution of the deposits’ volume \( D_t \) by assuming a linear relationship between the logarithm of the normalized deposits volume \( \lambda_t = \log(\frac{D_t}{D_0}) \) and a combination of the risk factors (or rather some functions of them), plus an autoregressive term. Thus, the general formula that describes the evolution of this function \( D_t \) is given by

\[
\lambda_t = \beta_0 + \beta_1 \lambda_{t-1} + \beta_2 X_{t}^{(1)} + \beta_3 X_{t}^{(2)} + \beta_4 X_{t}^{(3)} + \epsilon_t \tag{1}
\]

The risk factors \( X_{t}^{(1)}, X_{t}^{(2)}, X_{t}^{(3)} \) are functions of all the variables that define the movement of \( \lambda_t \): the market rate \( R_t \), the interest rate \( I_t \) and the CDS \( S_t \). \( \beta_0, \beta_1, \beta_2, \beta_3, \beta_4 \) are the coefficients and \( \epsilon_t \) is the error term.
2.2 Deposits Interest Rate

Being a tool of the pricing policy of the banks, the deposits’ interest rate is a driving factor that affects the evolution of deposits’ volume through time. An increase of the deposit rate will work as an incentive for existing depositors not to withdraw from their accounts or even to increase the amount deposited.

On the other hand, the deposits’ rates will be also dependent on the general level of the market interest rates and on the perceived creditworthiness of the bank: in both cases, higher values of the risk factors tend to increase the level of the interests paid on deposits.

As for the dependence between the deposits’ rates and these two risk factors, we present two alternatives: first, we assume a linear dependence of the interest rate from time moving averages of the 1M Eonia rates and the CDS index (linear model). As a second alternative, we use an Error Correction Model.

2.2.1 Linear model

The linear model for the deposits interest rate \( I_t \) has the following structure:

\[
I_t = \beta_{0,I} + \beta_{1,I} A(R_t, p) + \beta_{2,I} A(S_t, q) + \epsilon_t
\]  

(2)

where \( \beta_{0,I}, \beta_{1,I} \) and \( \beta_{2,I} \) are the coefficients of the linear dependence (to be established with a linear regression) and \( \epsilon_t \) is the error term (in the Appendix A we define explicitly the operator \( A(x, y) \)). \( S_t \) is a modification of the CDS index, introduced to consider its effects only when it is above a given threshold:

\[
S_t = \begin{cases} 
0 & \text{if } S(t) < 0.03 \\
S(t) & \text{otherwise}
\end{cases}
\]

The idea here is that when the perceived credit risk of the bank is small, it does not affect the deposit interest rate model. We found the 300 bps works quite well in capturing the historical data.

By analysing the ACF and PACF plots of the errors process \( \epsilon_t \) obtained from a preliminary ordinary least square estimation, we have found the error terms to be autocorrelated. So the most suitable model to describe the deposits interest rate \( I_t \) is an AR on residuals, estimated by Cochrane-Orcutt procedure. Using an AR(1) model the autocorrelation vanished, so the correct model for the error term is:

\[
\epsilon_t = \rho \epsilon_{t-1} + \varepsilon_t
\]

(3)

where now the independent random variable is \( \varepsilon_t \).

2.2.2 Error correction model (ECM)

The structure of the error correction model is given by the following system of linear equations:

\[
I^L_T = \beta_{0,L}^L + \beta_{1,L}^L R_t + \beta_{2,L}^L S_t + \epsilon_t
\]

\[
\Delta I^L_T = \beta_{0,L}^S + \beta_{1,L}^S \Delta R_t + \beta_{2,L}^S \Delta S_t + \beta_{3,L}^S (I_{t-1} - I^L_{t-1}) + \eta_t
\]

(4)

(5)

where \( \epsilon_t \) and \( \eta_t \) are error terms. It should be stressed that, differently form what happens in practice in many cases, our version of the ECM considers also the CDS index.
In the following we will assume that is not possible to have negative values for the deposits interest rate. This constraint can be clearly relaxed if one believes that the bank is able to charge negative rates to its clients.

### 2.3 Market Interest Rate

The relevant market interest rate for our models is a short-term risk-free rate: we consider the 1M Eonia swap rate as a good proxy for this rate. We choose to model the dynamic of the market interest rate with an extension of the CIR dynamics introduced by Cox, Ingersoll and Ross [5], denoted as CIR++, which is due to Brigo and Mercurio [3]. In this dynamics the instantaneous risk-free rate \( r_t \) is given by the sum of a deterministic time function component \( \psi_t \) and a stochastic variable \( x_t \), which follows a mean reverting square-root process, the original CIR dynamics:

\[
\begin{align*}
    r_t &= \psi_t + x_t, \\
    dx_t &= \kappa_x (\theta_x - x_t) dt + \sigma_x \sqrt{x_t} dW^x_t
\end{align*}
\]

where \( \kappa_x, \theta_x \) and \( \sigma_x \) are the parameters of the CIR dynamics and \( W^x \) represents the Wiener process.

Starting from this dynamics we can compute a zero-coupon bond price with maturity time \( T \) as follows:

\[
P(t, T) = e^{-\int_t^T \psi_s ds} A(t, T) e^{-B(t,T)x_t}
\]

where the functions \( A(t, T) \) and \( B(t, T) \) are defined as:

\[
A(t, T) = \left[ \frac{2\gamma \exp[(\kappa + \gamma)(T - t)/2]}{2\gamma + (\kappa + \gamma)(\exp[(T - t)\gamma] - 1)} \right]^{\frac{\nu}{2}}
\]

\[
B(t, T) = \frac{2 \exp[\gamma(T - t)] - 1}{2\gamma + (\kappa + \gamma)(\exp[\gamma(T - t)] - 1)}
\]

\[
\gamma = \frac{4\kappa \theta}{\sigma^2}, \\
\nu = \frac{4\kappa \theta}{\sigma^2}
\]

From (6) and (8) we can derive the simply compounded Eonia swap rate \( R(t_{i-1}, t_i) \) for a maturity time \( \Delta t \), which in our case is equal to \( \frac{30}{365} = 1 \) month. In a generic form, the simply compounded rate between time \( t_{i-1} \) and \( t_i \) valued at time \( t_0 \):

\[
R(t_{i-1}, t_i) = R_{t_{i-1}} = \left( \frac{1}{E[D(t_{i-1}, t_i)]} - 1 \right) \frac{1}{\Delta t}
\]

where \( D(s, t) = \exp(-\int_s^t r_u du) \) represents the discount factor up to time \( t \). Conditioned at the value \( r(s) \), in the CIR setting \( E[D(s, t)] = P(s, t) \); at time \( t_0 \), we have:

\[
R(t_0, t_1) = R_{t_0} = \left( \frac{1}{P(t_0, t_1)} - 1 \right) \frac{1}{\Delta t}
\]
2.4 Credit Default Swap

For the CDS index we choose not to use a credit model to evaluate derivative contracts; we will model the spread directly with a standard CIR dynamics:

\[
\frac{dS_t}{S_t} = \kappa_s (\theta_t - S_t) dt + \sigma_s \sqrt{S_t} W^s_t
\]

where \( \kappa_s, \theta_s \) and \( \sigma_s \) are the parameters of the model and \( W^s_t \) is the Wiener process.

3 Model Selection and Estimation

We select the factors that are supposed to affect the deposits’ dynamics for each cluster, trying to capture the different behaviour of depositors. For example, some depositors may be more concerned about the default of the bank than others, while some other depositors may just look at the interest paid by the bank. Hence, to model the deposits’ volume and interest rates, we aim at identifying a regression that best fits the historical data.

For all clusters, we consider the associated regressions acceptable only if the coefficients, resulting from the estimation, have the expected theoretical sign (for example, the deposits’ interest rate should be positively correlated with both the Eonia rate and the CDS index) and if the obtained estimators are statistically significant.

3.1 The Data Set

For the deposits’ volume and the deposits’ interest rate, the source of the data is the Statistical Database of Banca d’Italia.\(^2\) All data are quarterly and the period analysed ranges from January 2005 to June 2015.

In more detail, we retrieved the following time series:

- Volumes on sight current account deposits: classified by sector and size of deposit (code \([\text{TDB30960}_5540161]\)) for the products. Since the data set contains amounts multiplied by the number of days, we obtain the deposits’ volume by dividing the quarterly product by the actual number of days in each quarter.
- Interest rates on sight current account deposits: classified by sector and size of deposit (code \([\text{TDB30960}_5640113]\)) for the deposits’ interest rate.

We allocate the available data into 8 clusters, made of two types of depositors, each one subdivided in four ranges of deposit amount, as follows:

- Non financial corporations and producer households (NFC&PH),
  - up to 10.000 Euros;
  - from 10.000 Euros to 50.000 Euros;
  - from 50.000 Euros to 250.000 Euros;
  - more than 250.000 Euros.
- Consumer households, non profit institutions (NPIS) serving households and unclassifiable units (CH,NSH&UI),

\(^2\)The data are publicly available at https://www.bancaditalia.it/statistiche/basi-dati/bds/index.html.
Figure 3: Plots of the time series retrieved from the public data base of Banca d’Italia.

- up to 10,000 Euros;
- from 10,000 Euros to 50,000 Euros;
- from 50,000 Euros to 250,000 Euros;
- more than 250,000 Euros.

The plots of the time series of deposits’ volume and interest rate for each cluster are in Figure 3.

The 1M Eonia swap is collected daily data, from 1/1/2005 to 30/6/2015, from Bloomberg and a quarterly average has been compute. The plot of the time series is in Figure 4.

Finally, a CDS index is built as a proxy for the CDS of the Italian Banking Industry: we compute a quarterly weighted average of the CDS of the major Italian banks, where the weights are proportional to the sight deposits volumes in each bank's balance sheet at the reference date (June 2015). The banks considered are Banco Popolare, BPM, BNL, Intesa Sanpaolo, Mediobanca, Monte dei Paschi di Siena, UBI and Unicredit.

In more detail, we collected daily data, from 1/1/2005 to 30/6/2015, from Bloomberg and we computed a quarterly average for each bank. Then we computed
Figure 4: Eonia 1M quarterly average time series.

Figure 5: CDS quarterly time series for each bank and the CDS index for the system (in basis points).

the quarterly average for the system.

The Bloomberg tickers for the banks are:

- BPIIM CDS EUR SR 5Y D14 MSG1 Corp for Banco Popolare
- LAVORO CDS EUR SR 5Y D14 MSG1 Corp for BNL
- PMIIM CDS EUR SR 5Y D14 MSG1 Corp for BPM
- ISPIM SPA CDS EUR SR 5Y D14 MSG1 Corp for Intesa Sanpaolo
- BACRED CDS EUR SR 5Y D14 MSG1 Corp for Mediobanca
- UBIIM CDS EUR SR 5Y D14 MSG1 Corp for UBI
- UCGIM CDS EUR SR 5Y D14 MSG1 Corp for Unicredit

In Figure 5 we show the plot of the CDS quarterly time series for each bank and the CDS index for the system.
3.2 Model Selection Procedure

For each cluster we choose the most suitable model both considering the $R^2$ value and the p-values (0.05 being the chosen significance level). Additionally we perform a backtesting, by splitting the sample into two sub-samples and checking whether the model estimated within the first sub-sample remains sound within the second one.

More specifically, we perform two different tests:

- **Test on residuals**: We split the sample of size $n=72$ into two sub-samples of size $m$ and $n-m$ dates. We test whether, estimating a model within the first sample and using it on the second one, there is a significant difference in the size of the residuals between the two sub-samples.

- **Forward predictive failure test**: We split the sample of size $n$ into two sub-samples of size $m$ and $n-m$. We test whether there is a significant difference in the parameters in estimating a model for each sub-sample (using the same independent variables).

Details are in Appendix B.

3.3 Regression Analysis

We present for each of the 8 clusters the regressions resulting from the model selection for:

- the deposit interest rate, both the linear (LD) and Error Correction Model (ECM) type of dynamics;
- the deposits’ volume.

The details on the estimated coefficients and statistical robustness analysis are in Appendix C and D.

Non financial corporations and producer households, up to 10.000 Euros

**Up to 10.000 Euros**

- **Interest rate $I_t$ (LD):**
  \[ I_t = \beta_{0,1} + \beta_{1,1} A(R_t, 6) + \epsilon_t \]
  \[ \epsilon_t = \rho \epsilon_{t-1} + \eta_t \]

  In Figure 6a we plot the actual values of the time series.

- **Interest rate $I_t$ (ECM):**
  \[ I^{LT}_t = \beta_{0,1}^{LT} + \beta_{1,1}^{LT} R_t + \epsilon_t \]
  \[ \Delta I^{LT}_t = \beta_{0,1}^{ST} + \beta_{1,1}^{ST} \Delta R_t + \beta_{2,1}^{ST} (I_{t-1} - I^{LT}_{t-1}) + \eta_t \]

  In Figure 6b we plot the actual values of the time series of the interest rate and the fitted values obtained by using the model above and the estimated coefficients.

- **Deposits’ volume $D_t$:**
  \[ \lambda_t = \beta_0 + \beta_1 \lambda_{t-1} + \beta_2 A(I_t, 3) + \epsilon_t \]

  In Figure 7 we show the results of the fitted dynamics along with the actual values.
Figure 6: Non financial corporations and producer households up to 10.000 Euros: Time series of interest rate actual vs. fitted values for the period from 1/2005 to 6/2015.

Figure 7: Non financial corporations and producer households up to 10.000 Euros: Time series of deposits’ volume actual vs. fitted values for the period from 1/2005 to 6/2015 (in billions of Euros).

**From 10.000 to 50.000 Euros**

- **Interest rate** $I_t$ (LD):
  \[ I_t = \beta_{0,I} + \beta_{1,I}A(R_t, 6) + \epsilon_t \]
  \[ \epsilon_t = \rho \epsilon_{t-1} + \epsilon_t \]

  In Figure 8a we plot the actual values of the time series.

- for the **interest rate** $I_t$ (ECM):

  \[ I_t^{LT} = \beta_{0,I}^{LT} + \beta_{1,I}^{LT} R_t + \epsilon_t \]
  \[ \Delta I_t^{LT} = \beta_{0,I}^{ST} + \beta_{1,I}^{ST} \Delta R_t + \beta_{3,I}^{ST} (I_{t-1} - I_{t-1}^{LT}) + \eta_t \]

  In Figure 8b we plot the actual values of the time series of the interest rate and the fitted values obtained by using the model above and the estimated coefficients.

- **Deposits volume** $D_t$: 

Figure 8: Non financial corporations and producer households from 10.000 to 50.000 Euros: Time series of interest rate actual vs. fitted values for the period from 1/2005 to 6/2015.

Figure 9: Non financial corporations and producer households from 10.000 to 50.000 Euros: Time series of deposits’ volume actual vs. fitted values for the period from 1/2005 to 6/2015 (in billions of Euros)

Since we found seasonality on our data (whatever the trend is, the deposits volume drops systematically each year in the first quarter and then increases in the second one) we first employ a dummy variable $DV$ to seasonally adjust them. This variable is equal to one in the first quarter, equal to zero otherwise. Our equation is:

$$\lambda_t = \beta_0 + \beta_1 \lambda_{t-1} + \beta_2 A(I_t, 3) + \beta_3 DV_t + \epsilon_t$$

In Figure 9a we show the raw values $\lambda_t$, together with the seasonally adjusted values $\lambda'_t$.

We use the seasonally adjusted data for a second and final regression:

$$\lambda'_t = \beta_0 + \beta_1 \lambda'_{t-1} + \beta_2 A(I_t, 3) + \epsilon_t$$

In Figure 9b we show the fitted values along with the raw ones.

**From 50.000 to 250.000 Euros**
Figure 10: Non financial corporations and producer households from 10,000 to 50,000 Euros: Time series of interest rate actual vs. fitted values for the period from 1/2005 to 6/2015.

- for the **Interest rate** $I_i$ (LD):

$$I_t = \beta_{0,i} + \beta_{1,i} A(R_{t-1}, 3) + \beta_{2,i} A(\bar{S}_t, 12) + \epsilon_t$$

$$\epsilon_t = \rho \epsilon_{t-1} + \epsilon_t$$

In Figure 10a we plot the actual values of the time series.

- **Interest rate** $I_L$ (ECM):

$$I_{LT} = \beta_{0,L} + \beta_{1,L} R_t + \beta_{2,L} \bar{S}_t + \epsilon_t$$

$$\Delta I_{LT} = \beta_{0,L} + \beta_{1,L} \Delta R_t + \beta_{2,L} \Delta \bar{S}_t + \beta_{3,L} (I_{t-1} - I_{LT}^{t-1}) + \eta_t$$

In Figure 10b we plot the actual values of the time series of the interest rate and the fitted values obtained by using the model above and the estimated coefficients.

- **Deposits volume** $D_t$:

Since we found seasonality on our data (whatever the trend is, the deposits volume drops systematically each year in the first quarter and then increases in the second one) we first employ a dummy variable $DV$ to seasonally adjust them. This variable is equal to one in the first quarter, equal to zero otherwise. Our equation is:

$$\lambda_t = \beta_0 + \beta_1 \lambda_{t-1} + \beta_2 A(C_t, 6) + \beta_3 A(\bar{S}_t, 6) + \beta_4 DV_t + \epsilon_t$$

where $C_t = R_t - I - t$. In Figure 11a we show the seasonally adjusted values along with the raw ones.

We use the seasonally adjusted data for a second and final regression:

$$\lambda'_{t} = \beta_0 + \beta_1 \lambda'_{t-1} + \beta_2 A(C_t, 6) + \beta_3 A(\bar{S}_t, 6) + \epsilon_t$$

In Figure 11b we show the results of the fitted dynamics along with the actual values.
Figure 11: Non financial corporations and producer households from 50.000 to 250.000 Euros: Time series of deposits’ volume actual vs. fitted values for the period from 1/2005 to 6/2015 (in billions of Euros)

Figure 12: Non financial corporations and producer households more than 250.000 Euros: Time series of interest rate actual vs. fitted values for the period from 1/2005 to 6/2015.

**More than 250.000 Euros**

- **Interest rate \( I \) (LD):**
  \[
  I_t = \beta_{0,1}I + \beta_{1,1}A(R_{t-1}, 3) + \beta_{2,1}A(\bar{S}_t, 12) + \epsilon_t
  \]
  \[
  \epsilon_t = \rho \epsilon_{t-1} + \xi_t
  \]
  In Figure 12a we plot the actual values of the time series.

  - for the **interest rate \( I \) (ECM):**
    \[
    I_t^{LT} = \beta_{0,1}^{LT} + \beta_{1,1}^{LT}R_t + \beta_{2,1}^{LT}\bar{S}_t + \epsilon_t
    \]
    \[
    \Delta I_t^{LT} = \beta_{0,1}^{ST} + \beta_{1,1}^{ST}A(R_t) + \beta_{2,1}^{LT}\Delta\bar{S}_t + \beta_{3,1}^{ST}(I_{t-1} - I_{t-1}^{LT}) + \eta_t
    \]
  
  In Figure 12b we plot the actual values of the time series of the interest rate and the fitted values obtained by using the model above and the estimated coefficients.
Figure 13: Non financial corporations and producer households more than 250.000 Euros: Time series of deposits’ volume actual vs. fitted values for the period from 1/2005 to 6/2015 (in billions of Euros)

- **Deposits volume** $D_t$:

Since we found seasonality on our data (whatever the trend is, the deposits volume drops systematically each year in the first quarter and then increases in the second one) we first employ a dummy variable $DV$ to seasonally adjust them. This variable is equal to one in the first quarter, equal to zero otherwise. Our equation is:

$$\lambda_t = \beta_0 + \beta_1 \lambda_{t-1} + \beta_2 A(C_t, 6) + \beta_3 A(\bar{S}_t, 6) + \beta_4 DV_t + \epsilon_t$$

We seasonally adjust the data by subtracting them the value of the dummy variable multiplied by its coefficient. In Figure 13a we show the seasonally adjusted values along with the raw ones.

We use the seasonally adjusted data for a second and final regression:

$$\lambda'_t = \beta_0 + \beta_1 \lambda'_{t-1} + \beta_2 A(C_t, 6) + \beta_3 A(\bar{S}_t, 6) + \epsilon_t$$

In Figure 13b we show the results of the fitted dynamics along with the actual values.

Consumer households, NPIIS serving households and unclassifiable units, up to 10.000 Euros.

**Up to 10.000 Euros**

- **Interest rate** $I_t$ (LD):

$$I_t = \beta_{0,I} + \beta_{1,I} A(R_{t-1}, 3) + \epsilon_t$$

$$\epsilon_t = \rho \epsilon_{t-1} + \xi_t$$

In figure 14a we plot the actual values of the time series.

- **Interest rate** $I_t$ (ECM):
Figure 14: Consumer households, NPIS serving households and unclassifiable units, up to 10,000 Euros: Time series of deposits’ volume actual vs. fitted values for the period from 1/2005 to 6/2015.

![Observed data and fitted values](image1)

\[ I_t^{LT} = \beta_0^{LT} + \beta_1^{LT} R_t + \epsilon_t \]
\[ \Delta I_t^{LT} = \beta_0^{ST} + \beta_1^{ST} \Delta R_t + \beta_2^{ST} (I_{t-1} - I_{t-1}^{LT}) + \eta_t \]

In Figure 14b we plot the actual values of the time series of the interest rate and the fitted values obtained by using the model above and the estimated coefficients.

- Deposits volume \( D_t \):
  \[ \lambda_t = \beta_0 + \beta_1 \lambda_{t-1} + \beta_2 A(I_t, 3) + \epsilon_t \]

In Figure 15 we show the results of the fitted dynamics along the actual values.

**From 10,000 to 50,000 Euros**
- Interest rate \( I \) (LD):
Figure 16: Consumer households, NPIS serving households and unclassifiable units, from 10.000 to 50.000 Euros: Time series of interest rate actual vs. fitted values for the period from 1/2005 to 6/2015

\[ I_t = \beta_{0,I} + \beta_{1,I} A(R_{t-1}, 3) + \epsilon_t \]
\[ \epsilon_t = \rho \epsilon_{t-1} + \varepsilon_t \]

In Figure 16a we plot the actual values of the time series.

- **Interest rate** \( I_t \) (ECM):

\[ I_{t}^{LT} = \beta_{0,I}^{LT} + \beta_{1,I}^{LT} R_t + \epsilon_t \]
\[ \Delta I_{t}^{LT} = \beta_{0,I}^{ST} + \beta_{1,I}^{ST} \Delta R_t + \beta_{3,I}^{ST} (I_{t-1} - I_{t-1}^{LT}) + \eta_t \]

In Figure 16b we plot the actual values of the time series of the interest rate and the fitted values obtained by using the model above and the estimated coefficients.

- **Deposits volume** \( D_t \):

\[ \lambda_t = \beta_0 + \beta_1 \lambda_{t-1} + \beta_2 A(C_t, 3) + \beta_3 A(\bar{S}_t, 3) + \epsilon_t \]

In Figure 17 we show the results of the fitted values along with the observed ones.

**From 50.000 to 250.000 Euros**

- **Interest rate** \( I_t \) (LD):

\[ I_t = \beta_{0,I} + \beta_{1,I} A(R_{t-1}, 3) + \beta_{2,I} A(\bar{S}_t, 12) + \epsilon_t \]
\[ \epsilon_t = \rho \epsilon_{t-1} + \varepsilon_t \]

In figure 18a we plot the actual values of the time series.

- **Interest rate** \( I_t \) (ECM):

\[ I_{t}^{LT} = \beta_{0,I}^{LT} + \beta_{1,I}^{LT} R_t + \beta_{2,I}^{LT} S_t + \epsilon_t \]
\[ \Delta I_{t}^{LT} = \beta_{0,I}^{ST} + \beta_{1,I}^{ST} \Delta R_t + \beta_{2,I}^{ST} \Delta S_t + \beta_{3,I}^{ST} (I_{t-1} - I_{t-1}^{LT}) + \eta_t \]

In Figure 18b we plot the actual values of the time series of the interest rate and the fitted values obtained by using the model above and the estimated coefficients.
Figure 17: Consumer households, NPIS serving households and unclassifiable units, from 10,000 to 50,000 Euros: Time series of deposits’ volume actual vs. fitted values for the period from 1/2005 to 6/2015 (in billions of Euros).

Figure 18: Consumer households, NPIS serving households and unclassifiable units, from 50,000 to 250,000 Euros: Time series of interest rate actual vs. fitted values for the period from 1/2005 to 6/2015

- **Deposits volume** $D_t$:
  \[
  \lambda_t = \beta_0 + \beta_1 \lambda_{t-1} + \beta_2 A(R_t, 3) + \beta_3 A(I_t, 3) + \beta_4 A(\bar{S}_t, 6) + \epsilon_t
  \]
  In Figure 19 we show the results of the fitted dynamics along with the actual values.

**More than 250,000 Euros**
- **Interest rate** $I_t$ (LD):
  \[
  I_t = \beta_{0,I} + \beta_{1,I} A(R_t, 6) + \beta_{2,I} A(\bar{S}_t, 12) + \epsilon_t
  \]
  \[
  \epsilon_t = \rho \epsilon_{t-1} + \varepsilon_t
  \]
  In figure 20a we plot the actual values of the time series.
- **Interest rate** $I_t$ (ECM):
Figure 19: Consumer households, NPIS serving households and unclassifiable units, from 50,000 to 250,000 Euros: Time series of 3-months deposits volume vs. fitted values for the period from 1/2005 to 6/2015 (in billions of Euros).

Figure 20: Consumer households, NPIS serving households and unclassifiable units, more than 250,000 Euros: Time series of interest rate actual vs. fitted values for the period from 1/2005 to 6/2015.

\[
I_t^{LT} = \beta_0^{LT} + \beta_1^{LT} R_t + \beta_2^{LT} S_t + \epsilon_t
\]

\[
\Delta I_t^{LT} = \beta_0^{ST} + \beta_1^{ST} \Delta R_t + \beta_2^{ST} \Delta S_t + \beta_3^{ST} (I_{t-1} - I_{t-1}^{LT}) + \eta_t
\]

In Figure 20b we plot the actual values of the time series of the interest rate and the fitted values obtained by using the model above and the estimated coefficients.

- **Deposits volume** \( D_t \):

  \[
  \lambda_t = \beta_0 + \beta_1 \lambda_{t-1} + \beta_2 A(R_t, 3) + \beta_3 A(I_t, 3) + \beta_4 A(S_t, 6) + \epsilon_t
  \]

  In Figure 21 we show the results of the fitted dynamics along with the actual values.

### 3.4 Calibration of the Risk Factors’ Dynamics

We now describe in detail the procedures used to calibrate to real market data the variables 1M Eonia swap rate and the 5Y CDS index.
Figure 21: Consumer households, NPIS serving households and unclassifiable units, more than 250.000 Euros: Time series of deposits volume actual vs. fitted values for the period from 1/2005 to 6/2015 (in billions of Euros).

<table>
<thead>
<tr>
<th>κx</th>
<th>θx</th>
<th>σx</th>
<th>r0</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.0164</td>
<td>6.12%</td>
<td>4.314%</td>
<td>0.1%</td>
</tr>
</tbody>
</table>

Table 1: Calibrated parameters in the CIR model for Eonia.

Eonia Swap Rate

The CIR++ model to model the short rate is calibrated to the time series of Eonia swap rates sampled monthly from 29/02/2008 to 30/06/2015. For each month we have a term structure of Eonia swap rates retrieved from Bloomberg, made of the following maturities: all months from 1 to 11, all years from 1 to 12, then 15 and 20 years.

The calibration of the parameters has been performed in two steps:

- the parameters κx, θx and σx relative to the stochastic part xt have been estimated by using a Kalman filter calibration on our data and the starting point r0 is set equal to the last value of the time series, if it is positive, and equal to 0.1% otherwise. In our calibration the estimated parameters and the initial value are shown in Table 1.3

- the function ψt is determined by imposing an exact fit of the CIR++ model to the discount factors derived from Eonia swap rates’ term structure on the last date of the time series (end June 2015). The term structure is shown in Figure 22.

In Table 2 we show the calibrated values of the ψt function.

Credit Default Swap Index

The CIR model is calibrated to the time series of the CDS index by a procedure suggested by Brigo et al. Brigo et al. [2]. Details are in Appendix E.

3More details on the Kalman filter calibration of the CIR++ model can be found in Castagna and Fede [4].
Figure 22: Term structure of Eonia Swap rates at end June 2015, derived from market quotes for maturities running from 1 week to 10 years. Maturity are expressed as fraction year on the x-axis.

<table>
<thead>
<tr>
<th>Maturity</th>
<th>$\psi_t$</th>
<th>Maturity</th>
<th>$\psi_t$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1M</td>
<td>-0.00036</td>
<td>2Y</td>
<td>-0.00207</td>
</tr>
<tr>
<td>2M</td>
<td>-0.00051</td>
<td>3Y</td>
<td>-0.0016</td>
</tr>
<tr>
<td>3M</td>
<td>-0.00073</td>
<td>4Y</td>
<td>-0.00098</td>
</tr>
<tr>
<td>4M</td>
<td>-0.00098</td>
<td>5Y</td>
<td>-0.00031</td>
</tr>
<tr>
<td>5M</td>
<td>-0.00093</td>
<td>6Y</td>
<td>0.000313</td>
</tr>
<tr>
<td>6M</td>
<td>-0.00107</td>
<td>7Y</td>
<td>0.000543</td>
</tr>
<tr>
<td>7M</td>
<td>-0.00125</td>
<td>8Y</td>
<td>0.000391</td>
</tr>
<tr>
<td>8M</td>
<td>-0.00117</td>
<td>9Y</td>
<td>-0.00015</td>
</tr>
<tr>
<td>9M</td>
<td>-0.00173</td>
<td>10Y</td>
<td>-0.00028</td>
</tr>
<tr>
<td>10M</td>
<td>-0.00142</td>
<td>11Y</td>
<td>-0.00071</td>
</tr>
<tr>
<td>11M</td>
<td>-0.00154</td>
<td>12Y</td>
<td>-0.00193</td>
</tr>
<tr>
<td>1Y</td>
<td>-0.00212</td>
<td>15Y</td>
<td>-0.00485</td>
</tr>
<tr>
<td></td>
<td></td>
<td>20Y</td>
<td>-0.00804</td>
</tr>
</tbody>
</table>

Table 2: Values of the $\psi_t$ function.
Table 3: Coefficients in the CIR model for the CDS index.

\[
\begin{array}{ccc}
\kappa_s & \theta_s & \sigma_s \\
0.64559 & 2.1234\% & 13.51205\%
\end{array}
\]

The results of the calibration are given in Table 3. The last value of the historical time series is used as the starting value of the process in the simulation. So \(S_0 = 121.02\) bps.

4 Simulations

Once the parameters are estimated, we can run simulations for the evolution of each factor. We generate a number of simulated paths for the factors by means of the estimated dynamics, following these steps:

- compute 25,000 paths for the market rate 1M Eonia swap rate and for the CDS index, according to the CIR++ model and the CIR model respectively;
- for each path, compute the corresponding deposits’ interest rate and deposits' volume, according to the estimated regressions;
- we compute the value for every variable at each time step with a cut-off horizon \(T = 10\) years; each path is made of 120 steps of 1 month length.

We show the plots obtained from the simulations of deposits’ volumes for NFC&PH and CH,NSH&UI, respectively in Figures 23 and 24. We plot the expected evolution for each deposit cluster for the two types of depositors, departing from the last date of the historical time series; additionally, we plot the 99%, 99.5% and 99.9% percentile of the simulated paths at each future time step, to provide a visual indication of the volatility of the deposits’ volume.

5 Profitability and Risk Management

The approach we have sketched above and calibrated to historical data is a powerful tool to analyse under a unified framework the profitability and the risk related to sight deposits.

Until few years ago, before 2008, sight deposits were a very cheap liability for a bank. In many cases, the spread over the market short term risk-free interest rate was negative, meaning that the reinvestment of the volumes deposited by the bank’s customers earned a net profit.

In the current economic environment, and after the crisis of the banking sector in 2008/2009, the possibility for the banks to extract a positive net profit from the sight deposits dramatically decreased. This is due to a variety of reasons, amongst which we mention: i) the general raise of funding spreads for banks, which affected also the sight deposits since they are a relevant funding source; ii) the stricter regulatory rules for the monitoring of the liquidity risk, such as the limits of the LCR and NSFR ratios set by the Basel Committee, making more difficult the reinvestment in less liquid, but more profitable, assets; iii) the increase, in some instances, of a bank-run threat by depositors, which forced bank to raise the interest rate paid on deposits for a more or less long period of time.
Figure 23: Non financial corporations and producer households: Time series of the deposit’s volume and simulated values up to 10 years.

In any case, it is important to assess which are the deposit clusters that allow banks to have a higher profitability, with a forward looking view rather than a historical one. A profitability indicator is the Economic Value (EV) associated to a given cluster: it provides the present value of the NII, up the cut-off date, under the specific assumption that all the volume is allocated in a short-term rolling investment with a maturity equal to the repricing period of the deposits (1 month in our analysis).

Since we are not considering all the aspects related to the liquidity management, which may hinder or limit the simple rolling reinvestment, nor the compliance with the regulatory ratios, the EV should represent an upper boundary of the expected NII originated by the sight deposits. Moreover, we do not deal with the volatility of the EV, so we will not explicitly consider how to optimally allocate the deposits’ volume in fixed income investments with longer maturities, so as to lock in the received interests, satisfying the risk preferences of the bank about the liquidity risk and the probability of a liquidity shortage.

Nonetheless, we indirectly address the issue related to the volatility of the NII and of the deposits’ volume, and their allocation on longer maturity investments, by calculating, for each cluster, the Duration (Dur), the Weighted Average Life (WAL) and the Term Structure of Liquidity (TSL).

The Duration of the sight deposits is calculated according to the standard text-
Figure 24: Consumer households, NPIS serving households and unclassifiable units: Time series of the deposit’s volume and simulated values up to 10 years.

book definition: it provides the (expected) financial maturity for each cluster of deposits, so that it is possible to build a counterbalancing portfolio of assets. The Weighted Average Life is computed both for the expected and stressed evolution of the volumes: it is a measure of the their volatility and of the time within which the volumes will drop to zero. The Term Structure of Liquidity is also computed both for the expected and stressed evolution of the volumes: it gives a more detailed picture of the amount available at each future time step and, conversely, which is the amount of deposits that can flow out from the bank’s balance sheet. Therefore, the Term Structure of Liquidity is useful for liquidity management and for reinvestment purposes.

5.1 Economic Value

At time $t = 0$, considering a time horizon equals to $T$ (=10 years in our case), the Economic Value to the bank of the sight deposits is defined as

$$
EV(0,T) = E^Q \left[ \int_0^T D(0,s) D_s (R_s - I_s) ds \right]
$$

(12)

where $D(0,s) = \exp(-\int_0^s r_u du)$ represents the discount factor up to time $s$ and $D_s$ is the value of the deposit at time $s$. The $EV$ is the expected difference between
the market interest rate, at which deposits’ volume is reinvested, and the interest rate paid on the deposits. This quantity can be positive or negative depending on a number of factors related to the Bank, markets and behaviour of the depositors.

Since we are working in discrete time steps the equation (12) becomes:

\[
\text{EV}(0, T) = E^Q \left[ \sum_{i=0}^{t-1} D(t_0, t_{i+1}) D_t (R_t - I_t) \Delta t \right]
\]  

(13)

where \( \Delta t \) is the time-step chosen in the simulation (we have fixed \( \Delta t = 1/12 \)) and \( t_i \) is the \( i \)-th month in the period of time \([t_0, t_f] = [0, T]\). The discount factor is accordingly modified as \( D(0, t_{i+1}) = \exp(- \sum_{j=0}^{t_f} R_t \Delta t) \).

Equation (12) not only indicates how much the bank may expect to earn from the volume of deposits, but it shows also how to hedge the interest rate risk for the sight deposits. In an ideal world, at each instant \( s \) the deposits’ amount is fully invested in an instantaneously maturing risk-free deposit, yielding \( r_s \); if a fraction or the full amount is withdrawn by the depositors, the instantaneous maturity grants the availability to the bank of the necessary liquidity. The value of the deposits depend on the difference between the rate paid by the bank of the deposit and the rate earned on the reinvestment in the risk-free deposit.

In a discrete setting, where equation (13) is used to assess the value of the deposits, even if the time interval is short (e.g.: 1 month, as the time step we are working with), the bank has to measure and manage the liquidity risk caused by the mismatch due to the possible withdrawal by the depositors and the reinvestment locked until the end of the discretisation chosen period. To this end, the bank will provide for a liquidity buffer to cope with the requests of the depositors, given the bank’s risk preferences.

As mentioned above, we will not dwell in the present analysis on how the liquidity management will affect the \( \text{EV} \). For this reason, the results we will show should be considered as theoretical maximum values.

In Table 4 we show the values of the economic value for each different cluster, evaluated with both the linear and ECM dynamics for the deposit interest rate. It is worth noting that the \( \text{EV} \) of the deposits whose size is superior to 250,000 Euros is negative, because of the higher expected deposit interest rates paid to clients.

For ease of comparison, in Table we report the values of the Economic Value as a percentage of the amount deposited at the beginning of the simulation (June 2015). Unsurprisingly, the percentage \( \text{EV} \) has a negative correlation with the size of the deposits, both for the NFC&PH and the CH,NSH&UI.

Finally, it is interesting noting that the \( ECM \) dynamics for the deposit interest rates produces lower \( \text{EVs} \) than the linear dynamics. This is due to the lower explanatory power of the \( ECM \) dynamics in describing the evolution of the deposits’ rates, in their relationship with the 1M Eonia swap rate and the CDS index.

### 5.2 The Duration

The Duration of the liability associated to the sight deposits is defined as

\[
\text{Dur}^L = E^Q \left[ \frac{\sum_{i=0}^{I-1} t_{i+1} D(0, t_{i+1}) [\Delta D_{t_{i+1}} - I_t D_t \Delta t]}{\sum_{i=0}^{I-1} D(0, t_{i+1}) [\Delta D_{t_{i+1}} - I_t D_t \Delta t]} \right]
\]

(14)

where \( \Delta D_{t_{i+1}} \) is the variation of the value of the deposit between time \( t_i \) and \( t_{i+1} \)
Table 4: Economic value of the liability for each depositors’ clusters (in thousands of Euros).

<table>
<thead>
<tr>
<th>Cluster</th>
<th>LD</th>
<th>ECM</th>
</tr>
</thead>
<tbody>
<tr>
<td>NFC&amp;PH up to 10,000 Euros</td>
<td>166.077</td>
<td>154.066</td>
</tr>
<tr>
<td>NFC&amp;PH from 10,000 to 50,000 Euros</td>
<td>419.547</td>
<td>416.545</td>
</tr>
<tr>
<td>NFC&amp;PH from 50,000 to 250,000 Euros</td>
<td>319.678</td>
<td>292.703</td>
</tr>
<tr>
<td>NFC&amp;PH more than 250,000 Euros</td>
<td>-4,254.538</td>
<td>-4,855.654</td>
</tr>
<tr>
<td>CH,NSH&amp;UI up to 10,000 Euros</td>
<td>1,444.896</td>
<td>1,410.801</td>
</tr>
<tr>
<td>CH,NSH&amp;UI from 10,000 to 50,000</td>
<td>3,176.881</td>
<td>3,027.003</td>
</tr>
<tr>
<td>CH,NSH&amp;UI from 50,000 to 250,000</td>
<td>1,119.892</td>
<td>798.000</td>
</tr>
<tr>
<td>CH,NSH&amp;UI more than 250,000 Euros</td>
<td>-2,257.636</td>
<td>-2,327.567</td>
</tr>
</tbody>
</table>

Table 5: Economic value of the liability as a percentage of the initial deposit volume for each depositors’ clusters.

<table>
<thead>
<tr>
<th>Cluster</th>
<th>LD</th>
<th>ECM</th>
</tr>
</thead>
<tbody>
<tr>
<td>NFC&amp;PH up to 10,000 Euros</td>
<td>2.67%</td>
<td>2.48%</td>
</tr>
<tr>
<td>NFC&amp;PH from 10,000 to 50,000 Euros</td>
<td>2.06%</td>
<td>2.05%</td>
</tr>
<tr>
<td>NFC&amp;PH from 50,000 to 250,000 Euros</td>
<td>0.84%</td>
<td>0.77%</td>
</tr>
<tr>
<td>NFC&amp;PH more than 250,000 Euros</td>
<td>-3.70%</td>
<td>-4.23%</td>
</tr>
<tr>
<td>CH,NSH&amp;UI up to 10,000 Euros</td>
<td>3.16%</td>
<td>3.08%</td>
</tr>
<tr>
<td>CH,NSH&amp;UI from 10,000 to 50,000</td>
<td>2.31%</td>
<td>2.20%</td>
</tr>
<tr>
<td>CH,NSH&amp;UI from 50,000 to 250,000</td>
<td>0.80%</td>
<td>0.57%</td>
</tr>
<tr>
<td>CH,NSH&amp;UI more than 250,000 Euros</td>
<td>-3.59%</td>
<td>-3.70%</td>
</tr>
</tbody>
</table>

The interpretation of Dur is the usual given in financial analysis: it can be used to have an indication of the optimal investment of the deposits’ volume.

In Table 6 we show the value of Dur for each cluster of depositors: it has a negative correlation with the size of the deposits, both for the NFC&PH and the CH,NSH&UI, and in some cases it can even be greater to the cut-off period (10 years in our case) for the smallest sizes.

The sight deposits can be seen as a floating rate liability with a stochastic notional. The fact that the Duration is not very short, as expected in the case of a floating instrument, is due mainly to the stochastic notional. A unified framework, such as the one we are working in, can provide counter-intuitive, yet consistent results.

<table>
<thead>
<tr>
<th>Cluster</th>
<th>LD</th>
<th>ECM</th>
</tr>
</thead>
<tbody>
<tr>
<td>NFC&amp;PH up to 10,000 Euros</td>
<td>10.18</td>
<td>10.19</td>
</tr>
<tr>
<td>NFC&amp;PH from 10,000 to 50,000 Euros</td>
<td>10.11</td>
<td>10.10</td>
</tr>
<tr>
<td>NFC&amp;PH from 50,000 to 250,000 Euros</td>
<td>9.67</td>
<td>9.67</td>
</tr>
<tr>
<td>NFC&amp;PH more than 250,000 Euros</td>
<td>8.97</td>
<td>9.05</td>
</tr>
<tr>
<td>CH,NSH&amp;UI up to 10,000 Euros</td>
<td>10.21</td>
<td>10.22</td>
</tr>
<tr>
<td>CH,NSH&amp;UI from 10,000 to 50,000</td>
<td>9.92</td>
<td>9.92</td>
</tr>
<tr>
<td>CH,NSH&amp;UI from 50,000 to 250,000</td>
<td>9.93</td>
<td>10.19</td>
</tr>
<tr>
<td>CH,NSH&amp;UI more than 250,000 Euros</td>
<td>9.05</td>
<td>9.05</td>
</tr>
</tbody>
</table>

Table 6: Duration of the liability for each cluster of depositors.
## 5.3 The Weighted Average Life

The WAL of the deposits intuitively is the average time until one Euro of the initial deposit $D_{t_0}$ is withdrawn by the clients, hence in this way we calculate the average life of the money in the deposit. The expected WAL is defined as:

$$WAL^e = - \sum_{i=0}^{I-1} t_{i+1} \left( \frac{D_{t_{i+1}}^e - D_{t_i}^e}{D_{t_0}} \right)$$

where $D_{t_i}^e$ represents the simulated expected value of the deposit volume at time $t_i$.

If we need to measure the deposits’ WAL when they are exposed to stressed scenarios, since we have the distribution of the WAL, thanks to (15), we can calculate the value of the $\alpha$ percentile, which we denote by $WAL^{\alpha}$, where we set $\alpha$ equal to 99, 99.5 and 99.9.

The results each cluster of depositors is shown in Table 7 for the linear dynamics for the deposits’ interest rates, and in Table 8 for the ECM dynamics.

The deposit size has a positive correlation with the amount potentially withdrawn in stressed scenarios. The most sensitive clusters are the NFC&PH more than 250.000 and the CH,NSH&UI from 50.000 to 250.000 and more than 250.000, while the smallest clusters are almost insensitive. This can be explained by the fact that deposits whose size is greater than 100.000 Euros are not guaranteed by the Interbank Fund for the Protection of Deposits (FITD); hence such depositors could be more concerned than the others in smaller clusters if the stressed scenario led to the default of the bank.
5.4 Liquidity Risk Management

Given a distribution of the future evolution of the deposits’ volume, we can define the so called Term Structure of Liquidity (TSL$\alpha$) (see [4]): it is a time series of the lowest volume of deposits, with a given confidence level $\alpha$, at any given time. To define the TSL$\alpha$ we start with the concept of historical minimum of deposits: on any given scenario we can evaluate the quantity:

$$M(T) = \min_{0 \leq t \leq T} D(t)$$

where $D(t)$ is as usual the total volume of deposits. This number is the lowest level of deposits’ volume reached up to the time $T$, in a given scenario.

We can now consider various measures of this quantity over scenarios, in order to obtain stressed predictions on the future behaviour of deposits. For a percentile $\alpha$ of values taken from the simulations, we can calculate what is the minimum level, at a given time, reached. This brings us to the definition of the TSL$\alpha$:

$$\text{TSL}_{\alpha}(t) = \left[ M(t) \right]_{\alpha}$$

where $M$ is the minimum as defined in (16), and the expression $\left[ \cdot \right]_{\alpha}$ indicates that equation (16) as been computed by considering all the deposits volumes taken at the $\alpha$-percentile of the simulations.

Basically, the TSL at a given percentile $\alpha$, differs from the $\alpha$-percentile path deriving from the simulations in that it can never be increasing, but it can only slow downward or at best stay constant. In that way, we can be sure that the TSL will never be above the starting amount of deposits that the bank has on its balance sheet.

For this reason, the TSL is crucial in defining the bank’s investment policy of the deposits’ volumes. In fact, even if the sight deposits have an immediate contractual maturity, the behavioural model we have defined above allows to design several investment policies, by allocating a fraction of the volume in a long-term bond portfolio, with the requirement of meeting the liquidity needs of the bank at any given moment. The TSL would never indicate an amount to invest at a given maturity greater than the amount available to the bank at the reference date, i.e.: the date the analysis is operated.

The plots of the TSL$_{\alpha}(t)$, with $\alpha = 99.9\%$ are shown in Figures 25 and 26 for, respectively, the NFC&PH and the CH,NSH&UI depositors.

To have a more numerical grasp on the risk related to the decline of the amount of sight deposits, in Tables 9 and 10 we show the values of the TSL$_{\alpha}(t)$ after 5 years and after 9 years and 11 months$^4$ as a percentage of the amount deposited at the reference date, when the simulation begins (June 2015). We consider the expected TSL, and the TSL at the 99-th, the 99.5-th and the 99.9-th percentile.

The drop in the TSL values after 5 years and after 9 years and 11 months is more evident in the large size clusters, both for the expected value and for the other percentiles. On the contrary, smaller size clusters’ amounts, after 5 years or 9 years and 11 months, are much closer (or even equal for the expected TSL) to the initial ones.

$^4$Please keep in mind that the cut-off is 10 years, and at that date the deposits’ volume is arbitrarily set at zero, whence the calculation at 9 years and 11 months.
Figure 25: Non financial corporations and producer households: Time series of the deposits’ volume and TSL99.9.

6 Conclusion

We modelled the sight deposits in the Italian banking system, clustering depositors according to their type (Non financial corporations and producer households, Consumer households, NPIS serving households and unclassifiable units, and size of the deposit, assuming a dependency from a set of stochastic risk factors.

We initially estimated a model for the deposit rates assuming a dependency on 1M Eonia swap rate and a CDS index built on a weighted average of 6 Italian banks’ CDS spreads. For the estimation we used both a linear model and an ECM, though the first one proved always to be more effective than the second. We found that both for the Non financial corporations and producer households and for the Consumer households, NPIS serving households and unclassifiable units only the large size deposits are sensitive to the CDS index while the smaller ones depend exclusively on the 1M Eonia swap rate.

We estimated a model for the deposits’ volume assuming a dependency on deposit interest rates, 1M Eonia swap rate, CDS index and an autoregressive term. We found that both for the Non financial corporations and producer households and for the Consumer households, NPIS serving households and unclassifiable units the large size deposits’ volume are negatively correlated with the CDS index and positively correlated with the interest rate spread (deposit interest rate – 1M Eonia
Figure 26: Consumer households, NPIS serving households and unclassifiable units: Time series of the deposits’ volume and TSL\textsubscript{99.9}.

The Duration and the WAL are both found to be negatively correlated with the deposit size. The Duration is quite long, considering the short rate indexation of the sight deposits, in some cases even longer than the cut-off horizon of 10 years. Additionally, the bigger size clusters are the most sensitive to stressed scenarios in terms of amount potentially withdrawals, as shown by the WAL.

At the end of the 10-year cut-off horizon, the amounts deposited in small size...
accounts dependent only on the deposit interest rates, are expected to be roughly 3% to 4% higher than they were in June 2015 depending on the category. On the contrary, for the larger size classes dependent on the CDS index and on the interest rate spread, the final levels are expected to be approximately 2% to 14% lower than the initial ones, depending on the cluster of depositors. This is also mirrored by the different TSLs, showing a more volatile distribution for bigger size clusters.

The analysis showed what likely is widely known within banks, that is: small size deposits, with any type of depositors, are more profitable and stable than big size deposits. We tried to quantitatively assess the differences in profitability and volatility of volumes for the 8 clusters we identified. As a caveat, we would like to stress that the analysis refers to the entire Italian banking system and it can serve only as a guide to single banks, which should run a similar exercise on their own internal data to come up with more applicable results.
References


A Operators

In the main text above we have used the following operators:

- the difference operator $\Delta$, defined as:
  \[ \Delta x_t = x_t - x_{t-1} \] (18)

- the back-shift operator $B$, defined as:
  \[ B^k(x_t) = x_{t-k} \] (19)

- the simple moving average operator $A$ over a specified number of lags $p$, defined as:
  \[ A(x_t, p) = \frac{B_0(x_t) + B_1(x_t) + \ldots + B_{p-1}(x_t)}{p} = \frac{1}{p} \sum_{j=0}^{p-1} B_j(x_t) \] (20)

B Model Selection

To select the most suitable model, we perform two different tests:

- **Test on residuals**: We split the sample of size $n$ into two sub-samples of size $m$ and $n-m$. We test whether, estimating a model within the first sample and using it on the second one, there is a significant difference in the size of the residuals between the two sub-samples. Thus, we compute
  \[ F = \frac{\sum_{i=m+1}^{n} e_i^2 / (n - m - 1 - k)}{\sum_{i=1}^{m} e_i^2 / (m - 1 - k)} \] (21)
  and we compare it to a percentile of a Fisher-Snedecor distribution with $n - m - 1 - k$ and $n - 1 - k$ degrees of freedom, where $e_i$ is the $i$-th residual and $k$ is the number of parameters. With $\alpha = 0.05$ significance level, we have that there is some evidence that residuals are greater in the second sub-sample if
  \[ F > F_{0.05}(n - m - 1 - k, n - 1 - k) \] (22)
  This means that the model is not suitable for the whole sample and it could be even worse out of sample, so we discard it.

- **Forward predictive failure test**: We split the sample of size $n$ into two sub-samples of size $m$ and $n-m$. We test whether there is a significant difference in the parameters in estimating a model for each sub-sample (using the same independent variables). Thus, we have:
  \[ Y_t = X_{1,t} \beta_1 + \ldots + X_{k,t} \beta_k + \epsilon_t \] (23)
  for the first sub-sample, and
  \[ Y_t = X_{1,t} \alpha_1 + \ldots + X_{k,t} \alpha_k + \epsilon_t \] (24)
  for the second, where $\beta_i$ and $\alpha_i$ are the parameters for each sample.
  More compactly, we can rewrite the two equations as follows:
  \[ Y_t = X_{1,t} \beta_1 + \ldots + X_{k,t} \beta_k + 1_t X_{1,t} \theta_1 + \ldots + 1_t X_{k,t} \theta_k + \epsilon_t \] (25)
where

\[ 1_t = \begin{cases} 
0 & \text{if } t \leq m \\
1 & \text{otherwise}
\end{cases} \]

and \( \theta_i = \alpha_i - \beta_i \).

We estimate this last model. The standard F test for the significance of joint variables is asymptotically valid. Thus we compute

\[
F = \frac{(R^2 - R^2_*)/j}{(1 - R^2)/(n - 1 - k)}
\]

where \( R^2 \) is the value of the model considering all the variables, \( R^2_* \) is the value of the model without the dummy ones, \( k \) is the number of total variables and \( j \) is the number of dummy variables. The aim is to see whether removing the dummy variables we have a significant decrease in the \( R^2 \) value. So we compare \( F \) to a percentile of a Fisher-Snedecor distribution with \( j \) and \( n - 1 - k \) degrees of freedom.

Being \( \alpha = 0.05 \) the significance level, we have some evidence that \( R^2_* \) is significantly smaller than \( R^2 \) if

\[
F > F_{0.05}(j, n - 1 - k)
\]

This means the dummy variables are jointly statistically significant and the model is unstable.

C Results of the Regression Estimation

Non financial corporations and producer households, up to 10.000 Euros

- interest rate \( I_t \) (LD):

\[
I_t = \beta_{0,t} + \beta_{1,t} A(R_t, 6) + \epsilon_t
\]

\[
\epsilon_t = \rho \epsilon_{t-1} + \varepsilon_t
\]

Table 11 shows the coefficients of the regression, their \( p \)-values and the \( R^2 \) value. The interest rate is basically linked to a 6-month moving average of the 1M Eonia swap rate. The model is autoregressive term for the errors.

<table>
<thead>
<tr>
<th>Coefficients</th>
<th>p-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \beta_{0,t} )</td>
<td>0.0009</td>
</tr>
<tr>
<td>( \beta_{1,t} )</td>
<td>0.1646</td>
</tr>
<tr>
<td>( \rho )</td>
<td>0.9271</td>
</tr>
<tr>
<td>( R^2 )</td>
<td>0.9719</td>
</tr>
</tbody>
</table>

Table 11: Non financial corporations and producer households up to 10.000 Euros: estimated coefficients in the model for the interest rate (LD).

- Interest rate \( I_t \) (ECM):
$$I_{LT}^t = \beta_{0,LT} + \beta_{1,LT} R_t + \epsilon_t$$

$$\Delta I_{LT}^t = \beta_{0,ST} + \beta_{1,ST} \Delta R_t + \beta_{3,ST} (I_{t-1} - I_{LT}^{t-1}) + \eta_t$$

Table 12 shows the coefficients of the regression, their p-values and the $R^2$ value. The short term variation is positively linked to the 1M Eonia swap rate variations, although it is not statistically significant. For both the long term and short term dynamics we obtain lower $R^2$ values than the linear dynamic model one.

<table>
<thead>
<tr>
<th>Coefficients</th>
<th>p-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\beta_{0,LT}$</td>
<td>0.0012 0.0000</td>
</tr>
<tr>
<td>$\beta_{1,LT}$</td>
<td>0.1260 0.0000</td>
</tr>
<tr>
<td>$\beta_{0,ST}$</td>
<td>-0.0001 0.5143</td>
</tr>
<tr>
<td>$\beta_{1,ST}$</td>
<td>0.0304 0.5836</td>
</tr>
<tr>
<td>$\beta_{2,ST}$</td>
<td>-0.3997 0.0179</td>
</tr>
<tr>
<td>$R^2_{LT}$</td>
<td>0.7667</td>
</tr>
<tr>
<td>$R^2_{ST}$</td>
<td>0.5405</td>
</tr>
</tbody>
</table>

Table 12: Non financial corporations and producer households up to 10.000 Euros: estimated coefficients in the model for the interest rate (ECM).

- **Deposits’ volume $D_t$:**

$$\lambda_t = \beta_0 + \beta_1 \lambda_{t-1} + \beta_2 A(I_t, 3) + \epsilon_t$$

Table 13 shows the coefficients of the regression, their p-values and the $R^2$ value. Besides an autoregressive term, we have a dependency on a 3-month moving average of the deposit rate and the signs of the coefficients are as expected. We did not find any statistically significant dependency both on the interest rate spread and the CDS. We do not have significant autocorrelation between errors, so we use a standard OLS regression.

<table>
<thead>
<tr>
<th>Coefficients</th>
<th>p-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\beta_0$</td>
<td>-0.1220 0.0064</td>
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<tr>
<td>$\beta_1$</td>
<td>0.4848 0.0103</td>
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<tr>
<td>$\beta_2$</td>
<td>7.5670 0.0182</td>
</tr>
<tr>
<td>$R^2$</td>
<td>0.7840</td>
</tr>
</tbody>
</table>

Table 13: Non financial corporations and producer households up to 10.000 Euros: estimated coefficients in the model for the deposits volume.

Non financial corporations and producer households, from 10.000 to 50.000 Euros

- **Interest rate $I_t$ (LD):**

$$I_t = \beta_{0,I} + \beta_{1,I} A(R_t, 6) + \epsilon_t$$

$$\epsilon_t = \rho \epsilon_{t-1} + \epsilon_t$$

Table 14 shows the coefficients of the regression, their p-values and the $R^2$ value. The model has an autoregressive term for the errors.
Table 14: Non financial corporations and producer households from 10.000 to 50.000 Euros: estimated coefficients in the model for the interest rate (LD).

<table>
<thead>
<tr>
<th>Coefficients</th>
<th>p-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\beta_{0,t}$</td>
<td>0.0012</td>
</tr>
<tr>
<td>$\beta_{1,t}$</td>
<td>0.2393</td>
</tr>
<tr>
<td>$\rho$</td>
<td>0.9165</td>
</tr>
<tr>
<td>$R^2$</td>
<td>0.9815</td>
</tr>
</tbody>
</table>

- for the interest rate $I_t$ (ECM):

$$I_t^{LT} = \beta_{0,t}^{LT} + \beta_{1,t}^{LT} R_t + \epsilon_t$$

$$\Delta I_t^{LT} = \beta_{0,t}^{ST} + \beta_{1,t}^{ST} \Delta R_t + \beta_{2,t}^{ST} (I_{t-1} - I_{t-1}^{LT}) + \eta_t$$

In Table 15 we show the values of the coefficients of the regressions for the long term and short term dynamics, along with their p-values and the $R^2$ values. The short term variation is positively correlated to 1M Eonia swap rate variations, although coefficient is not statistically significant. For both the long term and short term dynamics we obtain lower $R^2$ values than the linear dynamic model one.

<table>
<thead>
<tr>
<th>Coefficients</th>
<th>p-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\beta_{0,t}^{LT}$</td>
<td>0.0014</td>
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<tr>
<td>$\beta_{1,t}^{LT}$</td>
<td>0.1869</td>
</tr>
<tr>
<td>$\beta_{0,t}^{ST}$</td>
<td>-0.0001</td>
</tr>
<tr>
<td>$\beta_{1,t}^{ST}$</td>
<td>0.0337</td>
</tr>
<tr>
<td>$\beta_{2,t}^{ST}$</td>
<td>-0.4770</td>
</tr>
<tr>
<td>$R^2_{LT}$</td>
<td>0.8054</td>
</tr>
<tr>
<td>$R^2_{ST}$</td>
<td>0.6255</td>
</tr>
</tbody>
</table>

Table 15: Non financial corporations and producer households from 10.000 to 50.000 Euros: estimated coefficients in the model for the interest rate (ECM).

- Deposits volume $D_t$:

Since we found seasonality on our data (whatever the trend is, the deposits volume drops systematically each year in the first quarter and then increases in the second one) we first employ a dummy variable $DV_t$ to seasonally adjust them. This variable is equal to one in the first quarter, equal to zero otherwise. Our equation is:

$$\lambda_t = \beta_0 + \beta_1 \lambda_{t-1} + \beta_2 A(I_t, 3) + \beta_3 DV_t + \epsilon_t$$

Table 16 shows the coefficients of the regression, their p-values and the $R^2$ value. Beside an autoregressive term and a dummy variable, we have a dependency on a 3-month moving average of the deposits’ rate. The deposits rate p-value is higher than the significance level but the signs of the coefficients are as expected. We did not find any statistically significant dependency both on the interest rate spread and the CDS.

We seasonally adjust the data by subtracting them the value of the dummy variable multiplied by its coefficient:

$$\lambda'_t = \lambda_t - \beta_3 DV_t$$
where $\lambda'_t$ is the seasonally adjusted value.

We use the seasonally adjusted data for a second and final regression:

$$\lambda'_t = \beta_0 + \beta_1 \lambda'_{t-1} + \beta_2 A(I_t, 3) + \epsilon_t$$

In Table 17 we show the coefficients of this final regression with associated $p$-values and $R^2$. No statistically significant dependency both on the interest rate spread and the CDS exist. We do not have significant autocorrelation between errors, so we use a standard OLS regression.

<table>
<thead>
<tr>
<th>Coefficients</th>
<th>p-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\beta_0$</td>
<td>0.0296</td>
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<tr>
<td>$\beta_1$</td>
<td>0.7248</td>
</tr>
<tr>
<td>$\beta_2$</td>
<td>1.5901</td>
</tr>
<tr>
<td>$R^2$</td>
<td>0.7802</td>
</tr>
</tbody>
</table>

Table 17: Non financial corporations and producer households from 10.000 to 50.000 Euros: estimated coefficients in the model for the deposits volume.

Non financial corporations and producer households, from 50.000 to 250.000 Euros

- for the Interest rate $I$ (LD):

$$I_t = \beta_{0,I} + \beta_{1,I} A(R_{t-1}, 3) + \beta_{2,I} A(S_t, 12) + \epsilon_t$$

$$\epsilon_t = \rho \epsilon_{t-1} + \varepsilon_t$$

Table shows the coefficients of the regression, their $p$-values and the $R^2$ value. The interest rate has a relation with the 1M Eonia swap rate and the CDS, their coefficients are statistically significant and we obtained a high $R^2$. Additionally, we have a high autocorrelation between error terms, so we include an autoregressive term for the errors.

- Interest rate $I_t$ (ECM):

$$I_t^{LT} = \beta_{0,I}^{LT} + \beta_{1,I}^{LT} R_t + \beta_{2,I}^{LT} S_t + \epsilon_t$$

$$\Delta I_t^{LT} = \beta_{0,I}^{ST} + \beta_{1,I}^{ST} \Delta R_t + \beta_{2,I}^{ST} \Delta S_t + \beta_{3,I}^{ST}(I_{t-1} - I_{t-1}^{LT}) + \eta_t$$

Table 19 shows the values of the coefficients of the regressions for the long term and short term dynamics, their $p$-values and the $R^2$ values. The long term interest
rate is positively linked to the 1M Eonia swap rate and the CDS. The short term variation is not significantly linked to the 1m Eonia swap rate variations. Moreover, the coefficient of the CDS variation has a different sign from the one expected, and it is not significant too. For both the long term and short term dynamics we obtain lower $R^2$ values than the linear dynamic model one.

Table 19: Non financial corporations and producer households from 50.000 to 250.000 Euros: estimated coefficients in the model for the interest rate (ECM).

- **Deposits volume $D_t$:**

Since we found seasonality on our data, we employ a dummy variable $DV$ to seasonally adjust them. This variable is equal to one in the first quarter, equal to zero otherwise. Our equation is:

$$\lambda_t = \beta_0 + \beta_1 \lambda_{t-1} + \beta_2 A(C_t, 6) + \beta_3 A(\bar{S}_t, 6) + \beta_4 DV_t + \epsilon_t$$

In Table 20 we show the coefficients of the regression with associated $p$-values and $R^2$. Beside an autoregressive term and to the dummy variable, we have a dependency on a 6-month moving average of the spread between market and deposits' interest rates, and a link with a 6-month moving average of the CDS. The signs of the coefficients are as expected.

We seasonally adjust the data by subtracting them the value of the dummy variable multiplied by its coefficient:

$$\lambda'_t = \lambda_t - \beta_4 DV_t$$

where $\lambda'_t$ is the seasonally adjusted value.

We use the seasonally adjusted data for a second and final regression:

$$\lambda'_t = \beta_0 + \beta_1 \lambda'_{t-1} + \beta_2 A(C_t, 6) + \beta_3 A(\bar{S}_t, 6) + \epsilon_t$$
<table>
<thead>
<tr>
<th>Coefficients</th>
<th>p-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\beta_0$</td>
<td>0.1078</td>
</tr>
<tr>
<td>$\beta_1$</td>
<td>0.6708</td>
</tr>
<tr>
<td>$\beta_2$</td>
<td>1.4187</td>
</tr>
<tr>
<td>$\beta_3$</td>
<td>-0.6827</td>
</tr>
<tr>
<td>$\beta_4$</td>
<td>-0.0769</td>
</tr>
<tr>
<td>$R^2$</td>
<td>0.8471</td>
</tr>
</tbody>
</table>

Table 20: Non financial corporations and producer households, from 50.000 to 250.000 Euros: estimated coefficients in the model for seasonally adjusting the deposits volume.

In Table 21 we show the coefficients of the second regression with associated $p$-values and $R^2$. The signs of the coefficients are as expected. We do not have significant autocorrelation between errors, so we use a standard OLS regression.

<table>
<thead>
<tr>
<th>Coefficients</th>
<th>p-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\beta_0$</td>
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</tr>
<tr>
<td>$\beta_1$</td>
<td>0.8423</td>
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<tr>
<td>$\beta_2$</td>
<td>0.8448</td>
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<tr>
<td>$\beta_3$</td>
<td>-0.6214</td>
</tr>
<tr>
<td>$R^2$</td>
<td>0.9140</td>
</tr>
</tbody>
</table>

Table 21: Non financial corporations and producer households from 50.000 to 250.000 Euros: estimated coefficients in the model for the deposits’ volume.

Non financial corporations and producer households, more than 250.000 Euros

- **Interest rate $I$ (LD):**

$$I_t = \beta_{0,I} + \beta_{1,I}A(R_{t-1}, 3) + \beta_{2,I}A(\bar{S}_t, 12) + \epsilon_t$$

$$\epsilon_t = \rho \epsilon_{t-1} + \varepsilon_t$$

In Table 22 we provide the values for the coefficients of this regression, their $p$-values and the $R^2$ value. The interest rate has a positive relation with the Eonia swap rate and the CDS, the coefficients are statistically significant and we obtained a high $R^2$. We include an autoregressive term for the errors.

<table>
<thead>
<tr>
<th>Coefficients</th>
<th>p-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\beta_{0,I}$</td>
<td>0.0048</td>
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<tr>
<td>$\beta_{1,I}$</td>
<td>0.7417</td>
</tr>
<tr>
<td>$\beta_{2,I}$</td>
<td>0.0604</td>
</tr>
<tr>
<td>$\rho$</td>
<td>0.8983</td>
</tr>
<tr>
<td>$R^2$</td>
<td>0.9820</td>
</tr>
</tbody>
</table>

Table 22: Non financial corporations and producer households more than 250.000 Euros: estimated coefficients in the model for the interest rate (LD).

- for the **interest rate $I_t$ (ECM):**
\begin{align*}
I_t^{LT} &= \beta_{0,I}^{LT} + \beta_{1,I}^{LT} R_t + \beta_{2,I}^{LT} S_t + \epsilon_t \\
\Delta I_t^{LT} &= \beta_{0,I}^{ST} + \beta_{1,I}^{ST} \Delta R_t + \beta_{2,I}^{LT} \Delta S_t + \beta_{3,I}^{ST} (I_{t-1} - I_{t-1}^{LT}) + \eta_t
\end{align*}

Table 23 shows the values of the coefficients of the regressions for the long term and short term dynamics, their p-values and the $R^2$ values. The short term variation has positive relation with short term 1M Eonia swap rate variation, but not with the CDS variations and their coefficients are not statistically significant.

<table>
<thead>
<tr>
<th>Coefficients</th>
<th>p-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\beta_{0,I}^{LT}$</td>
<td>0.0049 0.0000</td>
</tr>
<tr>
<td>$\beta_{1,I}^{LT}$</td>
<td>0.6177 0.0000</td>
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<tr>
<td>$\beta_{2,I}^{LT}$</td>
<td>0.1406 0.0000</td>
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<td>$\beta_{0,I}^{ST}$</td>
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<tr>
<td>$\beta_{1,I}^{ST}$</td>
<td>0.2656 0.2727</td>
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<tr>
<td>$\beta_{2,I}^{ST}$</td>
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<tr>
<td>$\beta_{3,I}^{ST}$</td>
<td>-0.5174 0.0061</td>
</tr>
</tbody>
</table>

| $R^2_{LT}$ | 0.8525 |
| $R^2_{ST}$ | 0.6688 |

Table 23: Non financial corporations and producer households more than 250.000 Euros: estimated coefficients in the model for the interest rate (ECM).

- for the **Deposits volume** $D_t$:

Since we found seasonality on our data, we use a dummy variable $DV$ to seasonally adjust them. This variable is equal to one in the first quarter, equal to zero otherwise. Our equation is:

$$\lambda_t = \beta_0 + \beta_1 \lambda_{t-1} + \beta_2 A(C_t, 6) + \beta_3 A(S_t, 6) + \beta_4 DV_t + \epsilon_t$$

In Table 24 we have the coefficients of the regression, the associated p-values and $R^2$. The signs of the coefficients are as expected.

<table>
<thead>
<tr>
<th>Coefficients</th>
<th>p-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\beta_0$</td>
<td>0.0965 0.0000</td>
</tr>
<tr>
<td>$\beta_1$</td>
<td>0.8169 0.0000</td>
</tr>
<tr>
<td>$\beta_2$</td>
<td>4.4236 0.0012</td>
</tr>
<tr>
<td>$\beta_3$</td>
<td>-1.3453 0.0051</td>
</tr>
<tr>
<td>$\beta_4$</td>
<td>-0.0801 0.0000</td>
</tr>
</tbody>
</table>

| $R^2$ | 0.9632 |

Table 24: Non financial corporations and producer households, more than 250.000 Euros: estimated coefficients in the model for seasonally adjusting the deposits volume.

We seasonally adjust the data by subtracting them the value of the dummy variable multiplied by its coefficient:

$$\lambda_t' = \lambda_t - \beta_4 DV_t$$

where $\lambda_t'$ is the seasonally adjusted value.
We use the seasonally adjusted data for a second and final regression:
\[ \lambda'_t = \beta_0 + \beta_1 \lambda'_{t-1} + \beta_2 A(C_t, 6) + \beta_3 A(S_t, 6) + \epsilon_t \]

Table shows the coefficients of the regression, the \( p \)-values and \( R^2 \). The signs of the coefficients are as expected. We do not have significant autocorrelation between errors, so we use a standard OLS regression.

<table>
<thead>
<tr>
<th>Coefficients</th>
<th>p-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \beta_0 )</td>
<td>0.0687</td>
</tr>
<tr>
<td>( \beta_1 )</td>
<td>0.8462</td>
</tr>
<tr>
<td>( \beta_2 )</td>
<td>3.8084</td>
</tr>
<tr>
<td>( \beta_3 )</td>
<td>-1.1523</td>
</tr>
<tr>
<td>( R^2 )</td>
<td>0.9673</td>
</tr>
</tbody>
</table>

Table 25: Non financial corporations and producer households more than 250,000 Euros: estimated coefficients in the model for the deposits’ volume.

**Consumer households, NPIS serving households and unclassifiable units, up to 10,000 Euros.**

- **Interest rate \( I \) (LD):**
  \[ I_t = \beta_{0,I} + \beta_{1,I} A(R_{t-1}, 3) + \epsilon_t \]
  \[ \epsilon_t = \rho \epsilon_{t-1} + \epsilon_t \]

In Table 26 we show the values of the coefficients of this regression, \( p \)-values and the \( R^2 \) value. We have a high autocorrelation between error terms, so we include an autoregressive term for the errors.

<table>
<thead>
<tr>
<th>Coefficients</th>
<th>p-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \beta_{0,I} )</td>
<td>0.0006</td>
</tr>
<tr>
<td>( \beta_{1,I} )</td>
<td>0.1345</td>
</tr>
<tr>
<td>( \rho )</td>
<td>0.5923</td>
</tr>
<tr>
<td>( R^2 )</td>
<td>0.9884</td>
</tr>
</tbody>
</table>

Table 26: Consumer households, NPIS serving households and unclassifiable units, up to 10,000 Euros: estimated coefficients in the model for the interest rate (LD).

- **Interest rate \( I_t \) (ECM):**
  \[ I_t^{LT} = \beta_{0,t}^{LT} + \beta_{1,t}^{LT} R_t + \epsilon_t \]
  \[ \Delta I_t^{LT} = \beta_{0,t}^{ST} + \beta_{1,t}^{ST} \Delta R_t + \beta_{3,t}^{ST} (I_{t-1} - I_{t-1}^{LT}) + \eta_t \]

Table 27 shows the values of the coefficients of the regressions for the long term and short term dynamics, their \( p \)-values and the \( R^2 \) values. The short term variation has positive relation with short term 1M Eonia rate variation, although its \( p \)-value is slightly higher than the significance level. For both the long term and short term dynamics we obtain lower \( R^2 \) values than the linear dynamic model one.
Table 27: Consumer households, NPIS serving households and unclassifiable units, up to 10,000 Euros: estimated coefficients in the model for the interest rate (ECM).

- **Deposits volume** $D_t$:

  $\lambda_t = \beta_0 + \beta_1 \lambda_{t-1} + \beta_2 A(I_t, 3) + \epsilon_t$

  Table 28 shows the results of the regression. The signs of the coefficients are as expected. We haven’t find any statistically significant dependency both on the interest rate spread and the CDS. We do not have significant autocorrelation between errors, so we use a standard OLS regression.

Table 28: Consumer households, NPIS serving households and unclassifiable units, up to 10,000 Euros: estimated coefficients in the model for the deposits’ volume.

### Consumer households, NPIS serving households and unclassifiable units, from 10,000 to 50,000 Euros

- **Interest rate** $I_t$ (LD):

  $I_t = \beta_{0,I} + \beta_{1,I} A(R_{t-1}, 3) + \epsilon_t$

  $\epsilon_t = \rho \epsilon_{t-1} + \epsilon_t$

  In Table 29 we provide the values if the coefficients of this regression, their $p$-values and the $R^2$ value. We include an autoregressive term for the errors.

Table 29: Consumer households, NPIS serving households and unclassifiable units, from 10,000 to 50,000 Euros: estimated coefficients in the model for the interest rate (LD).

- **Interest rate** $I_t$ (ECM):

  \[ R^2_{LT} = 0.9294 \]
  \[ R^2_{ST} = 0.8140 \]

  Table 27: Consumer households, NPIS serving households and unclassifiable units, up to 10,000 Euros: estimated coefficients in the model for the interest rate (ECM).
\[ I_{t}^{LT} = \beta_{0,t}^{LT} + \beta_{1,t}^{LT} R_{t} + \epsilon_{t} \]

\[ \Delta I_{t}^{LT} = \beta_{0,t}^{ST} + \beta_{1,t}^{ST} \Delta R_{t} + \beta_{3,t}^{ST} (I_{t-1} - I_{t-1}^{LT}) + \eta_{t} \]

Table 30 shows the values of the coefficients of the regressions for the long term and short term dynamics, their p-values and the \( R^2 \) values. The short term variation is positively linked short term 1M Eonia swap rate variation, although the coefficient is not statistically significant.

<table>
<thead>
<tr>
<th>Coefficients</th>
<th>p-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \beta_{0,t}^{LT} )</td>
<td>0.0011 0.0000</td>
</tr>
<tr>
<td>( \beta_{1,t}^{LT} )</td>
<td>0.1886 0.0000</td>
</tr>
<tr>
<td>( \beta_{0,t}^{ST} )</td>
<td>-0.0001 0.2432</td>
</tr>
<tr>
<td>( \beta_{1,t}^{ST} )</td>
<td>0.0293 0.5303</td>
</tr>
<tr>
<td>( \beta_{2,t}^{ST} )</td>
<td>-0.7149 0.001</td>
</tr>
<tr>
<td>( R^2_{LT} )</td>
<td>0.9053</td>
</tr>
<tr>
<td>( R^2_{ST} )</td>
<td>0.8008</td>
</tr>
</tbody>
</table>

Table 30: Consumer households, NPIS serving households and unclassifiable units, from 10.000 to 50.000 Euros: estimated coefficients in the model for the interest rate (ECM).

- **Deposits volume** \( D_t \):
  \[ \lambda_t = \beta_0 + \beta_1 \lambda_{t-1} + \beta_2 A(C_t, 3) + \beta_3 A(S_t, 3) + \epsilon_t \]

In Table 31 we show the coefficients of the regression with the associated p-values and \( R^2 \). The dependency on a 3-month moving average of the market-deposit rate spread (although its p-value is slightly higher than the significance level) and a correlation with a 3-month moving average of the CDS. The signs of the coefficients are as expected. We do not have significant autocorrelation between errors, so we use a standard OLS regression.

<table>
<thead>
<tr>
<th>Coefficients</th>
<th>p-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \beta_0 )</td>
<td>0.0405 0.0070</td>
</tr>
<tr>
<td>( \beta_1 )</td>
<td>0.8462 0.0000</td>
</tr>
<tr>
<td>( \beta_2 )</td>
<td>0.4516 0.0925</td>
</tr>
<tr>
<td>( \beta_3 )</td>
<td>-0.3811 0.0475</td>
</tr>
<tr>
<td>( R^2 )</td>
<td>0.9296</td>
</tr>
</tbody>
</table>

Table 31: Consumer households, NPIS serving households and unclassifiable units, from 10.000 to 50.000 Euros: estimated coefficients in the model for the deposits volume.

Consumer households, NPIS serving households and unclassifiable units, from 50.000 to 250.000 Euros

- **Interest rate** \( I \) (LD):
  \[ I_t = \beta_{0,t} + \beta_{1,t} A(R_{t-1}, 3) + \beta_{2,t} A(S_t, 12) + \epsilon_t \]

\[ \epsilon_t = \rho \epsilon_{t-1} + \epsilon_t \]

Table 32 provides the results of this regression. We include an autoregressive term for the errors.
Table 32: Consumer households, NPIS serving households and unclassifiable units, from 50.000 to 250.000 Euros: estimated coefficients in the model for the interest rate (LD).

- Interest rate $I_t$ (ECM):

$$I^{LT}_t = \beta_{0,LT} + \beta_{1,LT} R_t + \beta_{2,LT} \bar{S}_t + \epsilon_t$$

$$\Delta I^{LT}_t = \beta_{0,ST} + \beta_{1,ST} \Delta R_t + \beta_{2,ST} \Delta \bar{S}_t + \beta_{3,ST} (I_{t-1} - I^{LT}_{t-1}) + \eta_t$$

In Table 33 we show the results. The long term interest rate has a positive correlation with the 1M Eonia rate and the CDS. The short term variation has a positive relation with short term 1M Eonia rate variation although its coefficient is not statistically significant. In addition, the coefficient of the CDS variation has a different sign from the one expected, and it is not significant too.

- Deposits volume $D_t$:

$$\lambda_t = \beta_0 + \beta_1 \lambda_{t-1} + \beta_2 A(R_t, 3) + \beta_3 A(I_t, 3) + \beta_4 A(\bar{S}_t, 6) + \epsilon_t$$

In Table 34 we show the coefficients of the regression with associated $p$-values and $R^2$. The volume depends on a 3-month moving average of the market rate, on a 3-month moving average of the deposit rate and on a 6-month moving average of the CDS. The signs of the coefficients are as expected. We did not find autocorrelation between errors, so we use a standard OLS model.
Table 34: Consumer households, NPIS serving households and unclassifiable units, from 50,000 to 250,000 Euros: estimated coefficients in the model for the deposits volume.

\[ I_t = \beta_{0,I} + \beta_{1,I} A(R_t, 6) + \beta_{2,I} A(\bar{S}_t, 12) + \epsilon_t \]
\[ \epsilon_t = \rho \epsilon_{t-1} + \eta_t \]

Table 35 shows the results of the regression. While the 1M Eonia swap rate coefficient is statistically significant, the CDS one is slightly higher than the significance level. We include an autoregressive term for the errors.

Table 35: Consumer households, NPIS serving households and unclassifiable units, more than 250,000 Euros: estimated coefficients in the model for the interest rate (LD).

- **Interest rate** \( I_t \) (ECM):

\[ I_t^{LT} = \beta_{0,L}^{LT} + \beta_{1,L}^{LT} R_t + \beta_{2,L}^{LT} \bar{S}_t + \epsilon_t \]
\[ \Delta I_t^{LT} = \beta_{0,I}^{ST} + \beta_{1,I}^{ST} \Delta R_t + \beta_{2,I}^{ST} \Delta \bar{S}_t + \beta_{3,I}^{ST} (I_{t-1} - I_{t-1}^{LT}) + \eta_t \]

In Table 36 we show the values of the coefficients of the regressions for the long term and short term dynamics, their p-values and the \( R^2 \) values. In The long term interest rate has a positive relation to the 1M Eonia rate and the CDS. The short term variation has positive relation to short term 1M Eonia rate variation although its coefficient is not statistically significant. In addition, the coefficient of the CDS variation has a different sign from the one expected and it is not significant too.

- **Deposits volume** \( D_t \):

\[ \lambda_t = \beta_0 + \beta_1 \lambda_{t-1} + \beta_2 A(R_t, 3) + \beta_3 A(I_t, 3) + \beta_4 A(\bar{S}_t, 6) + \epsilon_t \]

Table 37 we show the coefficients of the regression with associated p-values and \( R^2 \). The volume depends on a 3-month moving average of the market rate, on a 3-month moving average of the deposit rate and on a 6-month moving average of the CDS. The signs of the coefficients are as expected. We did not find autocorrelation between errors, so we use a standard OLS model.
Table 36: Consumer households, NPIS serving households and unclassifiable units, more than 250,000 Euros: estimated coefficients in the model for the interest rate (ECM).

<table>
<thead>
<tr>
<th>Coefficients</th>
<th>p-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\beta_{LT0}$</td>
<td>-0.0050 0.0000</td>
</tr>
<tr>
<td>$\beta_{LT1}$</td>
<td>0.5240 0.0000</td>
</tr>
<tr>
<td>$\beta_{ST0}$</td>
<td>0.00988 0.0001</td>
</tr>
<tr>
<td>$\beta_{ST1}$</td>
<td>-0.001 0.6970</td>
</tr>
<tr>
<td>$\beta_{ST2}$</td>
<td>0.2334 0.1800</td>
</tr>
<tr>
<td>$\beta_{ST3}$</td>
<td>-0.4859 0.0018</td>
</tr>
</tbody>
</table>

$R^2_{LT} = 0.8545$

$R^2_{ST} = 0.6800$

Table 37: Consumer households, NPIS serving households and unclassifiable units, more than 250,000 Euros: estimated coefficients in the model for the deposits volume.

<table>
<thead>
<tr>
<th>Coefficients</th>
<th>p-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\beta_0$</td>
<td>0.0963 0.0210</td>
</tr>
<tr>
<td>$\beta_1$</td>
<td>0.8406 0.0000</td>
</tr>
<tr>
<td>$\beta_2$</td>
<td>-7.5618 0.0000</td>
</tr>
<tr>
<td>$\beta_3$</td>
<td>8.0813 0.0010</td>
</tr>
<tr>
<td>$\beta_4$</td>
<td>-2.0750 0.0000</td>
</tr>
</tbody>
</table>

$R^2 = 0.9819$

**D Autocorrelation, Normality and Model Stability Test Results**

**D.1 Autocorrelation**

To test the autocorrelation of residuals we use the Durbin-Watson test. Thus we compute the $d$ statistic:

$$d = \frac{\sum_{t=2}^{T} (e_t - e_{t-1})^2}{\sum_{t=1}^{T} e_t^2}$$

where $e_t$ is the residual at time $t$. We compute the p-value of the $d$ statistic of the null hypothesis that the residuals from the linear regression are uncorrelated. The alternative hypothesis is that there is autocorrelation among the residuals. Given a 0.05 significance level, we reject the null hypothesis if p-value $< 0.05$.

When we find residuals from the OLS regression to be autocorrelated we estimate the parameters using a Cochrane-Orcutt model, then we repeat the test on the Cochrane-Orcutt residuals to check whether autocorrelation has been effectively eliminated. The p-values of the $d$ statistics are shown in Tables 38 and 39, respectively for the deposits’ interest rates and the deposits’ volume.

**D.2 Normality**

To test the normality of residuals we use the Jarque-Bera test: we compute the $jb$ statistic

$$JB = \frac{n - k + 1}{6} \left( S^2 + \frac{1}{4} (C - 3)^2 \right)$$

where $n$ is the number of observations, $k$ is the number of parameters, $S$ is the sample skewness, and $C$ is the sample kurtosis.
where $S$ is the sample skewness, $C$ is the sample kurtosis and $k$ is the number of regressors. The null hypothesis is that residuals come from a normal distribution, the alternative one is that they don’t come from such a distribution. Given a 0.05 significance level, we reject the null hypothesis if $JB = 1$, while we do not have sufficient evidence to reject it if $JB = 0$.

The $JB$ statistics are shown in Tables 40 and 41, respectively for the deposits’ interest rates and the deposits’ volume.

### D.3 Model stability

To test the model stability, we used the test on residuals and the test on parameters stability explained in Appendix B. We consider the model unstable if none of the tests is passed by the model.

Since we found autocorrelation only in the interest rates regressions, the test on residuals has been performed on the Cochrane-Orcutt model for the deposits’

**Table 38:** P-value of the $d$ statistic for the deposit interest rates’ regressions.

<table>
<thead>
<tr>
<th>Cluster</th>
<th>OLS</th>
<th>CO</th>
</tr>
</thead>
<tbody>
<tr>
<td>NFC&amp;PH up to 10.000 Euros</td>
<td>0.0000</td>
<td>0.0002</td>
</tr>
<tr>
<td>NFC&amp;PH from 10.000 to 50.000 Euros</td>
<td>0.0000</td>
<td>0.0234</td>
</tr>
<tr>
<td>NFC&amp;PH from 50.000 to 250.000 Euros</td>
<td>0.0000</td>
<td>0.3183</td>
</tr>
<tr>
<td>NFC&amp;PH more than 250.000 Euros</td>
<td>0.0000</td>
<td>0.6286</td>
</tr>
<tr>
<td>CH,NSH&amp;UI up to 10.000 Euros</td>
<td>0.0000</td>
<td>0.3587</td>
</tr>
<tr>
<td>CH,NSH&amp;UI from 10.000 to 50.000</td>
<td>0.0000</td>
<td>0.6440</td>
</tr>
<tr>
<td>CH,NSH&amp;UI from 50.000 to 250.000</td>
<td>0.0000</td>
<td>0.7821</td>
</tr>
<tr>
<td>CH,NSH&amp;UI more than 250.000 Euros</td>
<td>0.0000</td>
<td>0.2527</td>
</tr>
</tbody>
</table>

**Table 39:** p-value of the $d$ statistic for the deposit volumes’ regressions.

<table>
<thead>
<tr>
<th>Cluster</th>
<th>OLS</th>
</tr>
</thead>
<tbody>
<tr>
<td>NFC&amp;PH up to 10.000 Euros</td>
<td>0.0937</td>
</tr>
<tr>
<td>NFC&amp;PH from 10.000 to 50.000 Euros</td>
<td>0.0791</td>
</tr>
<tr>
<td>NFC&amp;PH from 50.000 to 250.000 Euros</td>
<td>0.2006</td>
</tr>
<tr>
<td>NFC&amp;PH more than 250.000 Euros</td>
<td>0.0755</td>
</tr>
<tr>
<td>CH,NSH&amp;UI up to 10.000 Euros</td>
<td>0.2444</td>
</tr>
<tr>
<td>CH,NSH&amp;UI from 10.000 to 50.000</td>
<td>0.1150</td>
</tr>
<tr>
<td>CH,NSH&amp;UI from 50.000 to 250.000</td>
<td>0.5304</td>
</tr>
<tr>
<td>CH,NSH&amp;UI more than 250.000 Euros</td>
<td>0.8322</td>
</tr>
</tbody>
</table>

**Table 40:** $JB$ statistic for the interest rates’ regressions.

<table>
<thead>
<tr>
<th>Cluster</th>
<th>OLS</th>
<th>CO</th>
</tr>
</thead>
<tbody>
<tr>
<td>NFC&amp;PH up to 10.000 Euros</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>NFC&amp;PH from 10.000 to 50.000 Euros</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>NFC&amp;PH from 50.000 to 250.000 Euros</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>NFC&amp;PH more than 250.000 Euros</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>CH,NSH&amp;UI up to 10.000 Euros</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>CH,NSH&amp;UI from 10.000 to 50.000</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>CH,NSH&amp;UI from 50.000 to 250.000</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>CH,NSH&amp;UI more than 250.000 Euros</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

Table 41: $JB$ statistic for the interest rates’ regressions.
Cluster & OLS
NFC&PH up to 10,000 Euros & 0
NFC&PH from 10,000 to 50,000 Euros & 0
NFC&PH from 50,000 to 250,000 Euros & 0
NFC&PH more than 250,000 Euros & 0
CH,NSH&UI up to 10,000 Euros & 1
CH,NSH&UI from 10,000 to 50,000 & 0
CH,NSH&UI from 50,000 to 250,000 & 0
CH,NSH&UI more than 250,000 Euros & 0

Table 41: JB statistic for the deposit volumes’ regressions.

<table>
<thead>
<tr>
<th>Cluster</th>
<th>F</th>
<th>$F_{0.05}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>NFC&amp;PH up to 10,000 Euros</td>
<td>0.25</td>
<td>2.33</td>
</tr>
<tr>
<td>NFC&amp;PH from 10,000 to 50,000</td>
<td>1.01</td>
<td>2.33</td>
</tr>
<tr>
<td>NFC&amp;PH from 50,000 to 250,000</td>
<td>0.89</td>
<td>2.42</td>
</tr>
<tr>
<td>NFC&amp;PH more than 250,000 Euros</td>
<td>1.94</td>
<td>2.42</td>
</tr>
<tr>
<td>CH,NSH&amp;UI up to 10,000 Euros</td>
<td>0.50</td>
<td>2.33</td>
</tr>
<tr>
<td>CH,NSH&amp;UI from 10,000 to 50,000</td>
<td>0.71</td>
<td>2.33</td>
</tr>
<tr>
<td>CH,NSH&amp;UI from 50,000 to 250,000</td>
<td>1.59</td>
<td>2.42</td>
</tr>
<tr>
<td>CH,NSH&amp;UI more than 250,000 Euros</td>
<td>1.38</td>
<td>2.42</td>
</tr>
</tbody>
</table>

Table 42: Values of $F$ and of $F_{0.05}$ for the backtest on residuals of the deposit interest rates’ regressions.

interest rates and on the OLS model for the deposits’ volume. The forward predictive failure test instead has been performed always on OLS.

The tested models for each cluster are those reported in Appendix C. The significance level is 0.05 and we set $n = 41$ and $m = 32$ while $k$ (and so $F_{0.05}$) depends on the depositors’ cluster. The results for test on residuals are given in Tables 42, 43; the results for forward prediction test are given in Tables 44 and 45.

<table>
<thead>
<tr>
<th>Cluster</th>
<th>F</th>
<th>$F_{0.05}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>NFC&amp;PH up to 10,000 Euros</td>
<td>0.77</td>
<td>2.42</td>
</tr>
<tr>
<td>NFC&amp;PH from 10,000 to 50,000</td>
<td>1.33</td>
<td>2.42</td>
</tr>
<tr>
<td>NFC&amp;PH from 50,000 to 250,000</td>
<td>2.48</td>
<td>2.53</td>
</tr>
<tr>
<td>NFC&amp;PH more than 250,000 Euros</td>
<td>1.76</td>
<td>2.53</td>
</tr>
<tr>
<td>CH,NSH&amp;UI up to 10,000 Euros</td>
<td>11.33</td>
<td>2.42</td>
</tr>
<tr>
<td>CH,NSH&amp;UI from 10,000 to 50,000</td>
<td>5.77</td>
<td>2.53</td>
</tr>
<tr>
<td>CH,NSH&amp;UI from 50,000 to 250,000</td>
<td>2.52</td>
<td>2.76</td>
</tr>
<tr>
<td>CH,NSH&amp;UI more than 250,000 Euros</td>
<td>1.86</td>
<td>2.76</td>
</tr>
</tbody>
</table>

Table 43: Values of $F$ and of $F_{0.05}$ for the back-test on residuals of the deposit volume’s regressions.
Table 44: Values of \( F \) and of \( F_{0.05} \) for the forward predictive failure test on the deposit interest rates’ regressions (see also Appendix B).

<table>
<thead>
<tr>
<th>Cluster</th>
<th>( F )</th>
<th>( F_{0.05} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>NFC&amp;PH up to 10.000 Euros</td>
<td>19.57</td>
<td>4.08</td>
</tr>
<tr>
<td>NFC&amp;PH from 10.000 to 50.000 Euros</td>
<td>24.51</td>
<td>4.08</td>
</tr>
<tr>
<td>NFC&amp;PH from 50.000 to 250.000 Euros</td>
<td>1.07</td>
<td>3.23</td>
</tr>
<tr>
<td>NFC&amp;PH more than 250.000 Euros</td>
<td>5.91</td>
<td>3.23</td>
</tr>
<tr>
<td>CH,NSH&amp;UI up to 10.000 Euros</td>
<td>1.00</td>
<td>4.08</td>
</tr>
<tr>
<td>CH,NSH&amp;UI from 10.000 to 50.000</td>
<td>1.50</td>
<td>4.08</td>
</tr>
<tr>
<td>CH,NSH&amp;UI from 50.000 to 250.000</td>
<td>1.96</td>
<td>3.23</td>
</tr>
<tr>
<td>CH,NSH&amp;UI more than 250.000 Euros</td>
<td>3.36</td>
<td>3.23</td>
</tr>
</tbody>
</table>

Table 45: Values of \( F \) and of \( F_{0.05} \) for the forward predictive failure test on the deposit volume’s regressions (see also Appendix B).

<table>
<thead>
<tr>
<th>Cluster</th>
<th>( F )</th>
<th>( F_{0.05} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>NFC&amp;PH up to 10.000 Euros</td>
<td>0.23</td>
<td>3.23</td>
</tr>
<tr>
<td>NFC&amp;PH from 10.000 to 50.000 Euros</td>
<td>1.12</td>
<td>3.23</td>
</tr>
<tr>
<td>NFC&amp;PH from 50.000 to 250.000 Euros</td>
<td>0.25</td>
<td>2.84</td>
</tr>
<tr>
<td>NFC&amp;PH more than 250.000 Euros</td>
<td>0.23</td>
<td>2.84</td>
</tr>
<tr>
<td>CH,NSH&amp;UI up to 10.000 Euros</td>
<td>5.21</td>
<td>3.23</td>
</tr>
<tr>
<td>CH,NSH&amp;UI from 10.000 to 50.000</td>
<td>0.35</td>
<td>2.84</td>
</tr>
<tr>
<td>CH,NSH&amp;UI from 50.000 to 250.000</td>
<td>1.75</td>
<td>2.61</td>
</tr>
<tr>
<td>CH,NSH&amp;UI more than 250.000 Euros</td>
<td>1.08</td>
<td>2.61</td>
</tr>
</tbody>
</table>

E  Details on the Calibration of the CDS Index Dynamics

The CIR model is calibrated to the time series of the CDS index by a procedure suggested by Brigo et al. [2]. The dynamic of the process can be written as:

\[
dS_t = \kappa_S (\theta_S - S_t)dt + \sigma_S \sqrt{S_t} dW^S_t
\]  (31)

where \( \kappa_S, \theta_S, \sigma_S > 0 \) and \( W^S_t \) is the Wiener process.

To ensure that \( S_t \) is always strictly positive one needs to impose the Feller condition:

\[ \sigma^2_S \leq 2\kappa_S \theta_S. \]

The simulation of the CIR process can be done using a recursive discrete version of the stochastic differential equation (31) with discretised time steps \( t_i \):

\[ S_{t_i} = \kappa_S \theta_S \Delta t + (1 - \kappa_S \Delta t) S_{t_{i-1}} + \sigma_S \sqrt{S_{t_{i-1}}} \Delta t \epsilon_i \]

with error terms \( \epsilon_i \) following a Gaussian distribution.

For CIR processes we have the following exact conditional distribution:

\[ f(S_{t_i}|S_{t_{i-1}}) = ce^{-u-v} \left( \frac{v}{u} \right)^{\frac{q}{2}} I_q(2\sqrt{uv}) \]

where

\[ c = \frac{2\kappa_S}{\sigma^2_S(1 - e^{-\kappa_S \Delta t})} \]

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\[ u = cS_t e^{-\kappa S \Delta t} \]
\[ v = cS_{t-1} \]
\[ q = \frac{2\kappa S \theta_S}{\sigma^2_S} - 1 \]

with \( I_q \) the modified Bessel function of the first kind of order \( q \) and \( \Delta t = t_i - t_{i-1} \).

To calibrate this model we apply the maximum likelihood estimation with starting parameters equal to:
\[ \kappa_0 = -\frac{\log(b)}{\Delta t} \]
\[ \theta_0 = \mathbb{E}(S_t) \]
\[ \sigma_0 = \frac{2b \text{VaR}(S_t)}{\theta_0} \]

where \( b \) is the coefficient obtained from a standard OLS estimation on our data.