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Funding, Liquidity, Credit and Counterparty Risk: Links and Implications

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1 Introduction

The problem of how to include correctly liquidity costs into the pricing of financial contracts has recently risen. The connections between the funding costs and the adjustments due to the compensation that a party has to pay to the counterparty for the losses on a contract caused by its default (the so called debit value adjustment, hereon **DVA**) have been inquired in some works (for example, Morini and Prampolini [10]). The related issue on how to properly compute and consider the **DVA** has been investigated by other authors (see for example Gregory [8] and Brigo and Capponi [3]).

In this work we try and clarify what is the essence of the **DVA**: we believe we offer a robust conceptual framework to consistently include the **DVA** in the balance sheet of a financial institution. Under this perspective, to our knowledge never proposed before, the **DVA** does not manifest any counterintuitive effects, such as a reduction of the current value of the liabilities of a counterparty when its creditworthiness worsens. On the other hand, the link between funding costs and **DVA** will be easily identified and considered, and in this way we can also establish in a thorough fashion how to discount positive and negative future cash-flows.

2 The Axiom

To derive a consistent theory of the links existing amongst funding and liquidity costs and counterparty and credit risks, we need to state an axiom that hopefully can be considered also sensible and widely accepted:

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Axiom 2.1. *Like in every human economic activity (by the very definition of the adjective "economic"), stockholders of a bank aim at making profits out of their investments in the business activity. As such they evaluate projects on the base of the profits, costs and the expected profit margin to be shared at the end of the bank's activity.*

The end of the bank can be indefinite, so that profits are shared out periodically: this is what usually happens in reality, where profits are computed and distributed on annual basis. Alternatively the end of the bank activity can be voluntarily set at a given date.

It is worth noticing that default is not included in the definition of voluntary end of activity, although the definition does not exclude the fact that default can be a rational option in some circumstances. In this case the decision to declare bankruptcy aims at minimizing losses and not at sharing (hopefully maximized) profits, absent upon a default's occurrence.

Axiom 2.1 has been sometimes named *going-concern* (see Fries [7] as an example).

3 Cash-flows Fair Values and Discounting

We start with considering a simple loan contract, (*e.g.*: a term deposit in the inter-bank market). Assume there are two economic operators (*e.g.*: two banks), B and L, where the first would like to borrow money from the second. To keep things simple, let us assume that there exist a constant risk-free interest r and that each operator pays a funding spread s_X, $X \in \{B, L\}$ over the the risk-free rate when borrowing money.[1]

The funding spread can be decomposed in two parts: i) a premium that is required by the lender for the default probability of the borrower, which we indicate by π_X, and the loss given default $\mathbf{Lgd}_X$ (expressed as a fraction of the lent amount), and ii) a possible liquidity premium γ_X (we still have $X \in \{B, L\}$).

At time $t = 0$, operator B asks operator L for a loan whose amount returned at the maturity T is K. L wants to price in the contract the risks and costs born, so as to make it fair (we assume that L does not want to earn any profit margin from the entire operation) and then to determine the amount P_L that can be lent, which makes the contract fair at inception. The present value of K at time T is its discounted value at the rate r if the counterparty B survives, otherwise, if it goes bankrupt, it is the present value of the recovery $(1 - \mathbf{Lgd})K$; to further lighten the notation, we assume without much loss of generality that $\mathbf{Lgd} = 100\%$, so that the recovery is 0. We assume for the moment that $\gamma_X = 0$, so that $s_X = \pi_X$ for either

[1]The present work admittedly moves from the "groundwork" established by Morini and Prampolini [10], trying to clarify some points and to correct some flaws. As such, the notation is the same as in their work, whenever possible, so as to make the comparison easier.

parties.[2] We will relax both assumptions later on.

We have to sum the present value of the costs[3] that the lender L has to pay: L has to fund the amount P and the future funding cost is the difference between the amount he/she has to pay back $P_L e^{(r+s_L)T}$ and the same amount invested at the risk-free rate $P_L e^{rT}$.[4] Summing up these components, we have that the amount P_L that L can lend to B at time 0 is obtained by making the value of the deal V_L at inception nil:

$$\begin{aligned} V_L = & -P_L + e^{-rT}[K - K\mathbb{E}[1 - \mathbf{1}_{\tau_B > T}] - P_L\big(e^{(r+s_L)T} - e^{rT}\big)] = \\ & - P_L e^{s_L T} + K e^{-(r+\pi_B)T} = 0 \end{aligned} \tag{1}$$

The fair amount lent will be then $P_L = Ke^{-(r+s_L+\pi_B)T}$, or

$$P_L = Ke^{-(r+s_L+s_B)T}$$

since we assumed the liquidity premium equal to 0.

Apparently it seems that L has to discount positive cash-flow received in T at a discount rate which include the risk-free rate, its own funding spread and the borrower's funding spread. Actually this is an effective rate that can be used to determine the fair amount to lend, but it is more useful, in our opinion, to consider the discount rate as just the risk-free rate, and then use this to discount expected cash-flow and costs. In fact, it is interesting to rewrite (1) in the following way:

$$V_L = -P_L + e^{-rT}K - \mathbf{CVA}_B - \mathbf{FC}_L \tag{2}$$

where $\mathbf{CVA}_B = e^{-rT}K\mathbb{E}[1 - \mathbf{1}_{\tau_B > T}]$ is the credit value adjustment due to the loss given default of B, in this case equal to the entire amount times the probability of default; $\mathbf{FC}_L = e^{-rT}P_L\big(e^{(r+s_L)T} - e^{rT}\big)$ is the funding cost born by the lender. The fair amount P_L is easily recognized as the present value received at T, minus the expected losses on default and minus the funding costs: $e^{-rT}K - \mathbf{CVA}_B - \mathbf{FC}_L$.

In Morini and Prampolini [10], the funding costs take into account the probability of default of L: when the lender goes bankrupt, he/she will not return the

[2] The fact that under the hypthesis that $\mathbf{Lgd} = 100\%$, $s_X = \pi_X$, is a consequence of a reduced form approach which we are modelling deafult risk within. For a general treatment of the reduced form approach to default modelling, see Duffie et al. [6].

[3] Costs have negative values, so they are algebraically added.

[4] The fact we are considering an investment in a risk free asset is not a loss of generality, since we can always adjust the expected cash-flows for the default risk and then consider these adjusted cash-flows invested at the risk-free rate.

compounded amount P to the funder, so that $\mathbf{FC}_L = e^{-rT} P_L \big(e^{(r+s_L)T} - e^{(r+s_L)T}(1 - e^{-\pi_L T}) - e^{rT} \big)$. Under the current assumption that $\gamma_L = 0$, the funding costs would be then nil.

In our setting, given axiom 2.1, it is not possible for the lender to consider his/her own default. Besides, we humbly believe that it is very unlikely for the bank's management to support with some success the argument before stockholders that they do not have to worry if they are not transferring funding costs in the pricing of their loans, because they will make up for all these extra costs when the bank goes burst. And this is not because it is not true that the bank will repay only a fraction of its debt on default, but simply because this is a false saving of money, since not paying back one's obligation means that no more equity is left. So the saving on the repayment of the debt should more correctly be seen as a loss on the obligor's equity. This will be clearer in what follows.

Let us now see how the borrower B price the loan contract. Basically he/she prices the contract with the same principles as the lender, so that the fair amount P_B that he/she should receive, should equal the present value of K, plus the funding costs and the $\mathbf{CVA}_L$ for the losses suffered if the lender declare bankruptcy. In a loan contract $\mathbf{CVA}_L$ is zero, since the borrower has no exposure to the lender, but only an obligation. So we can write:

$$V_B = P_B - e^{-rT}[K - P_B \big(e^{(r+s_B)T} - e^{rT} \big)] = -P_B e^{s_B T} + K e^{-rT} = 0 \tag{3}$$

The fair amount to B is then $P_B = K e^{-(r+s_B)T}$, which is different from the amount fair to the lender. The latter include also lenders' funding costs, whereas they are not considered in the valuation process by the borrower. Also in this case, it seems that negative cash flows should be discounted at an effective rate equal to the risk-free rate plus the borrower's spread, but this is just a way to set the fair level of the borrowed amount. Actually it is more consistent, in our view, to use just one rate, the risk-free one, to discount expected cash-flows and costs. In fact, recalling that $\gamma_B = 0$, we can write (3) as:

$$V_B = P_B - e^{-rT}[K - P_B \big(e^{(r+\pi_B)T} - e^{rT} \big)] = -P_B e^{\pi_B T} + K e^{-rT} = 0 \tag{4}$$

and hence $P_B = e^{-rT} K - \mathbf{DVA}_B$, where $\mathbf{DVA}_B = \mathbf{CVA}_B = e^{-rT} K (1 - e^{-\pi_B T})$ is the debit value adjustment, or the expected loss the borrower will cause to the lender on the occurrence of his/her default. In a loan contract the credit and value adjustments for B's default risk are the same. Besides $\mathbf{DVA}_B$ can be seen also as $\mathbf{FC}_B$, or the funding cost that the borrower has to pay: we will dwell more on this later on.

It is easy to check that $P_B - P_L = \mathbf{FC}_L$. This means that no agreement can be reached by the two counterparties in the loan contract, since the fair amount the borrower requires is higher than that one the lender is willing to lend. In other words the borrower's fair amount does not include the lender's funding costs.

This is apparently surprising, but actually it is not so far from what happened in reality in the last few years, starting from 2007, when the banks' funding spreads dramatically increased and the ability to close loans' deals with counterparties worsened. Indeed, if the borrower has an easy access to the capital market and he/she is able to ask for funds directly to investors, the intermediation of the banking system is neither required nor efficient. Investors are economic operators investing their capital without (or with a small) leverage, so that they do not include funding costs in their evaluation process. In this case is possible to have an investors' fair value which clashes with the borrower's, since they will only consider the $\mathbf{CVA}_B = \mathbf{DVA}_B$ in their capital allocation decisions.

An agreement can be reached between a lender who operates with funding (*e.g.*: a bank) and a borrower only if the latter does not have direct access to the capital market, so that he/she will consider the lender's funding cost has unavoidable. In this case $P_L = P_B$.

The main result of this section is that the choice of the discounting rate for positive and negative cash-flows poses no problems even in presence of default risk premium and funding costs, when these are taken into account in a consistent manner. Actually the discounting rate is only and always the risk-free one. It is used to discount expected cash-flows, expected losses given the counterparty's default and the funding costs. The fact that when deriving the fair amount in a loan deal we use effective discount rates given by the sum of the risk-free, credit spread and funding spread in case, is misleading: the focus should not be on identifying the right discount rate for different cases, but it should be shifted on identifying the expected cash-flows and costs that may occur during the duration of the contract.

Following this route we totally bypass the problem of the choice of the discount rate, dealt in Fries [7]: the author here introduces an hedging argument for future cash-flows and then he consistently derives the proper discount rates. We think that the proposed argument does not take into account that each cash-flow is not some abstract entity falling into the books of a financial institutions, requiring a hedging strategy whose costs entail a specific discount rate. Cash-flows, instead, are always originated within a specific contract implying costs, revenues, and risks. These must be accounted to calculate the value of the contract and cash-flows related to them have to be discounted with the risk-free rate. Incidentally we would like to notice here that the the attribute "risk-free" is quite superfluous and it is usually used since in practice (and often also in theory) effective rates are introduced which encompass many risks. If one wants to be rigorous, there is only one (possibly stochastic) interest rate that makes possible to determine how much one unit of the numeraire good (*i.e*: money) is worth at future times.

4 Critique of the Debit Value Adjustment

A great debate is currently open over the debit value adjustment and its treatment in banks' balance sheets. We try and analyse what **DVA** really means, by introducing also an accounting perspective, since we believe it adds to the understanding of the issue.

We assume that the borrower B is a bank with a very simplified balance sheet that it is marked to the market.[5] Mark-to-market is operated by discounting all expected and risk-adjusted cash-flows at the risk free rate r, as shown in the previous section. The stockholders decide to start the activity with an equity E and to stop it after a period of time T; the amount E is deposited in a bank account D_1, which we assume risk-free; besides they require no premium over the risk-free rate, so that it is also the hurdle rate to value investment projects. We assume also that no liquidity premium is paid by the borrower so that $s_B = \pi_B$.

4.1 Single Period Case

4.1.1 Time 0

At time 0, the bank closes a loan contract with a lender (*e.g.*: an institutional investor) which is not charging any funding cost when setting the fair amount to lend. The amount is deposited in a bank account D_2, also risk-free to avoid immaterial complications at the moment. The balance sheet at time 0 looks like as follows:

Assets	**Liabilities**
$D_1 = E$	$L = Ke^{-rT}$
$D_2 = Ke^{-(r+s_B)T}$	$-\mathbf{DVA}_B(0) = -e^{-rT}K(1-e^{-\pi_B T}) = -e^{-rT}K(1-e^{-s_B T})$
	E

The assets and liabilities balance and the $\mathbf{DVA}_B(0)$ is deducted from the risk-free present value of the loan paying back K in T: in this way the present value of the loan is exactly matching the amount of cash deposited in D_2, so that the deal generates no P&L (profits/losses) at inception.

Subtracting the **DVA** from the current value of the risk-free present value of the liabilities is generally how the debit value adjustment is included in the balance sheet; this common practice brings the rather disturbing consequence that when the creditworthiness of B worsens (*i.e.*: π_B ($= s_B$ in our case) increases), the present value of the liabilities declines: something counterintuitive that has been justified by several arguments generally not particularly convincing. Some banks in the last few years benefited from this situation given the current concept of **DVA**, basically seen simply as the **CVA** that counterparty prices in the contract, considered from the obligor's perspective. We believe instead that, given axiom 2.1, the **DVA** is something different, as we hope to completely prove in what follows.

We suggest that the **DVA** is not a reduction in the value of the liabilities due to the credit risk of the borrower, but it is actually the present value of the costs (or losses, if you wish) that the borrower has to pay due to the fact that he/she is not a risk-free economic operator, under axiom 2.1. When the **DVA** is considered as the negative of the **CVA**, it still keeps its notion of compensation for the counterparty

[5] This is not always true in reality but, at least as far as the trading book is concerned, this is a fair assumption.

risk, but this notion is valid only for the lender. When moving to the borrower's perspective, the negative of the **CVA**, *i.e.*: the **DVA**, modifies its nature from that of a compensation for a risk to that of a cost. The deduction from the liabilities can be justified by the compensation nature of the **DVA** that, given axiom 2.1, cannot be supported since stockholders do not consider their bank's default in the investments' evaluation process. If this holds true, the **DVA**, being a cost, has to be moved in the balance sheet to reduction of the value of the net equity, rather than of the risk-free present value of the debt, so that the balance sheet should read as:

Assets	**Liabilities**
$D_1 = E$	$L = Ke^{-rT}$
$D_2 = Ke^{-(r+s_B)T}$	
	——————— -
	E
	$-\mathbf{DVA}_B(0) = -Ke^{-rT}(1 - e^{-\pi_B T}) = -Ke^{-rT}(1 - e^{-s_B T})$

The assets and liabilities are still balancing but now we have a completely different picture of the balance sheet, since the deal produces a P&L at inception: a loss equal to the **DVA**. We now have to prove that the **DVA** is actually the present value of the costs born by the borrower until the expiry of the loan and the end of the borrowers activity.

4.1.2 Time T

Let us check what happens at time T: all the bank accounts earn the risk free rate and this is also true for the risk-free value of the debt; the $\mathbf{DVA}(T)$ collapses to 0, since the debt expiries. Eventually we have:

Assets	**Liabilities**
$D_1 = Ee^{rT}$	$L = K$
$D_2 = Ke^{-s_B T}$	
	——————— -
	E

The balance sheet is clearly not balancing since we are missing the profits and losses realized over the period $[0, T]$. In fact we have interests' income from the account D_1 (II_1) and the losses (ℓ) on the funding spread given by the difference of what is the final value of the the account D_2 and what is paid back on the loan:

$$\begin{aligned}(Ee^{rT} - E) + (Ke^{-s_B T} - K) &= E(e^{rT} - 1) - K(1 - e^{-s_B T}) \\ &= II_1 - \ell\end{aligned}$$

so that if we add to the equity E also the profits and losses, and we consider the outflow of cash to pay back the loan, the assets and liabilities are balancing again:

Assets	Liabilities
$D_1 = Ee^{rT}$	$L = 0$
$D_2 = Ke^{-s_B T} - K$	
	——————————-
	E
	$+II_1$
	$-\ell$

The borrower's activity is then closed and we value its profitability by including also the hurdle rate:

$$(Ee^{rT} - E) + (Ke^{-s_B T} - K) - E(e^{rT} - 1) = -K(1 - e^{-s_B T}) = -\ell$$

so that the entire activity generated a loss ℓ equal to the funding spread on the amount K.

The terminal balance sheet confirms also the correctness of our suggestion to consider the **DVA** the value of the losses suffered at the end of the loan rather than a reduction of the risk-free present value of the loan. In fact it is easy to check that realized losses are the compounded **DVA**: $\ell = \mathbf{DVA}(0)e^{rT}$.

4.2 Multi-Period Case

4.2.1 Time 0

We now would like to generalize the analysis to a multi-period setting, by assuming that the bank's activity spans over the interval $[0, 2T]$ made of 2 periods T: we will strike a balance in 0, at an intermediate time T and at the end of the activity $2T$. We have the same set-up as above and this time the bank asks for a loan maturing in $2T$, when it has to pay back the amount K. The balance sheet at time 0 is:

Assets	Liabilities
$D_1 = E$	$L = Ke^{-r2T}$
$D_2 = Ke^{-(r+s_B)2T}$	
	——————————-
	E
	$-\mathbf{DVA}_B(0) = -Ke^{-r2T}(1 - e^{-s_B 2T})$

The **DVA**(0) is now the present value of the costs paid at $2T$, and they are reducing the value of the equity E, following our definition of the debit value adjustment.

4.2.2 Time T

At time T interests accrue on the bank accounts and on the loan. The interests earned on D_1 are $II_1 = E(e^{rT} - 1)$; on D_2 interests are $II_2 = Ke^{-s_B 2T}(e^{-rT} - e^{-r2T})$; the loan interests are $II_L = K(e^{-(r+s_B)T} - e^{-(r+s_B)2T})$. The total is shown in the new balance sheet below, where also the updated **DVA** is included:

Assets	Liabilities
$D_1 = Ee^{rT}$	$L = Ke^{-rT}$
$D_2 = Ke^{-rT-s_B 2T}$	
	E
	$II_1 = E(e^{rT} - 1)$
	$-\mathbf{DVA}_B(T) = -Ke^{-rT}(1 - e^{-s_B T})$
	$-\ell(0,T) = -Ke^{-rT}(e^{-s_B T} - e^{-s_B 2T})$

The equity now is incremented by the interests II_1 earned on the first bank account, and it is decreased by the debit value adjustment at time T, $\mathbf{DVA}_B(T)$, and of the value of the amount of losses that can be attributed to the period $[0,T]$, $\ell(0,T) = II_2 - II_L$. It is very interesting to notice that:

$$-\mathbf{DVA}_B(0)e^{rT} = -\mathbf{DVA}_B(T) - \ell(0,T)$$

so that the balance sheet above can be re-written in totally equivalent way as:

Assets	Liabilities
$D_1 = Ee^{rT}$	$L = Ke^{-rT}$
$D_2 = Ke^{-rT-s_B 2T}$	
	E
	$II_1 = E(e^{rT} - 1)$
	$-\mathbf{DVA}_B(0)e^{rT} = -Ke^{-rT}(1 - e^{-s_B 2T})$

This choice of book keeping stresses the fact that also in multi-period setting, the value of the loss is still the **DVA** of the operation computed at the contracts' inception, compounded at each period with the risk-free rate. The first choice above, on the other hand, by isolating the losses, allows for their attribution to each period. This is true also with variable (possibly stochastic) spreads and interest rates.

4.2.3 Time 2T

Let us see what happens at $2T$, when the loan expiries and the bank (*i.e.*: the borrower) closes the business. In this case we have again the interests' accrual as in T, while the **DVA** is nil and the losses have to be updated to include also those referring to the second period:

Assets	Liabilities
$D_1 = Ee^{2rT}$	$L = 0$
$D_2 = Ke^{-s_B 2T} - K$	
	E
	$II_1 = E(e^{2rT} - 1)$
	$-\ell(0,T) = -K(e^{-s_B T} - e^{-s_B 2T})$
	$-\ell(T,2T) = -K(1 - e^{-s_B 2T})$

Once more it is quite easy to check that the assets and liabilities balance. It is also interesting to notice that:

$$-\ell(0,T) - \ell(T,2T) = -K(1-e^{-s_B 2T}) = \mathbf{DVA}(0)e^{r2T}$$

which confirms what we have stated above, that the total losses over the contract period are the future value of the **DVA** computed at the start of the contract and that the funding spread (and the risk-free rate) can also evolve stochastically until the maturity, since eventually only the initial level of the spread is what really counts. The evolution of the funding spread matters only in the attribution of portions of the total funding costs to a given period, something that is definitely important for practical accounting purposes.

We calculate also in this case the profitability of the bank's activity during its life, considering the hurdle rate for the invested capital, thus getting:

$$(Ee^{r2T} - E) + (Ke^{-s_B 2T} - K) - E(e^{r2T} - 1) = -K(1-e^{-s_B 2T}) = -\ell(0,2T)$$

so that the starting equity invested has been eroded by the total funding costs.

The analysis we have conducted clearly shows that the question whether to consider or not into the balance sheet the **DVA**, since it apparently generates perverse effects, is actually ill-posed. In reality the **DVA** is not a reduction of the current value of the liabilities, but it is simply the present value of the costs that a counterparty has to pay to compensate the other parties for the fact that it is not risk-free. Given axiom 2.1, the same amount is seen as a cost from the borrower's perspective, and as a default risk's compensation from the lender's perspective.

Seen as a the present value of a cost, the **DVA** is the reduction of the equity that can be determined from the start of the contract, although its monetary manifestation may occur only at the maturity. As such, it can be included in the (marked-to-market) balance sheet in a consistent fashion as a reduction of the equity, and no perverse effects manifests if the creditworthiness of the borrower worsens, since the present value of the costs increase and the net equity is accordingly abated. Under this perspective, the **DVA must** be included into the balance sheet without any doubt, thus fulfilling the *sound and prudent management* accounting principle.

4.3 DVA as Funding Benefit

As mentioned above several, not totally satisfying, justifications for the reduction of the liabilities produced by the **DVA**, have been provided in recent works. Gregory [8] presents a list of arguments on how to manage and monetize **DVA**, along with related pros and cons. He warns however on the very delicate nature of the **DVA**'s inclusion in a balance sheet.

Morini and Prampolini [10] argue that actually the **DVA** can be seen as a funding benefit, thus apparently fully justifying its insertion in the balance sheet as a reduction of the current value of the liabilities. However the argument it is not justified thoroughly and the consequences derive more from a *petitio principii* than from an in-depth analysis.

Let us check anyway if this argument impairs somehow our notion of **DVA**. Assume we are in a multi-period case, and we are at time T: at this moment the borrower asks for more funds to the lender, starting a new loan contract for an amount $K_2 < K$, to be paid back at time $2T$, together with the other loan K. K_2 is deposited in a (risk-free) bank account D_3. The balance sheet at time T, with the updated **DVA**, now reads as:

Assets	**Liabilities**
$D_1 = Ee^{rT}$	$L = Ke^{-rT}$
$D_2 = Ke^{-rT-s_B 2T}$	$L_2 = K_2 e^{-rT}$
$D_3 = K_2 e^{-rT-s_B T}$	
	E
	$II_1 = E(e^{rT} - 1)$
	$-\mathbf{DVA}_B(T) = -Ke^{-rT}(1 - e^{-s_B T}) - K_2 e^{-rT}(1 - e^{-s_B T})$
	$-\ell(0,T) = -Ke^{-rT}(e^{-s_B T} - e^{-s_B 2T})$

Morini and Prampolini (implicitly) suggest that the cash should not be deposited in a bank account (D_3 in our example), but it should be used to buy back some debt, thus reducing the funding need. Nothing prevents the implementation of this strategy, so that balance sheet, after the buying back of a portion of the first loan, is:

Assets	**Liabilities**
$D_1 = Ee^{rT}$	$L = (K - K_2)e^{-rT}$
$D_2 = Ke^{-rT-s_B 2T}$	$L_2 = K_2 e^{-rT}$
	E
	$II_1 = E(e^{rT} - 1)$
	$-\mathbf{DVA}_B(T) = -(K - K_2)e^{-rT}(1 - e^{-s_B T}) - K_2 e^{-rT}(1 - e^{-s_B T})$
	$-\ell(0,T) = -Ke^{-rT}(e^{-s_B T} - e^{-s_B 2T})$

the $\mathbf{DVA}_B(T)$ is reduced accordingly to the reduction of the debt whose original amount was K. The balance sheet at the end of the activities $2T$ is:

Assets	**Liabilities**
$D_1 = Ee^{2rT}$	$L = 0$
$D_2 = Ke^{-s_B T} + (K - K_2)e^{-s_B T}$	
$-(K - K_2) - K_2$	
	E
	$II_1 = E(e^{2rT} - 1)$
	$-\ell(0,T) = -K(e^{-s_B T} - e^{-s_B 2T})$
	$-\ell(T,2T) = -(K - K_2)(1 - e^{-s_B T}) - K_2(1 - e^{-s_B 2T})$

It is quite easy to check that the total loss is simply:

$$-\ell(0,T) - \ell(T,2T) = -K(1 - e^{-s_B 2T}) = \mathbf{DVA}(0)e^{r2T}$$

or, the cost paid on the total amount borrowed $K+K_2$, considering a buying back of the debt K_2, which leaves a total outstanding debt K equal to the staring amount. So, if we define a funding benefit as a reduction of the funding cost for a given amount of raised funds, we can easily see that given the net total amount funded over the period (K in our case), there is no reduction of the cost, which remains exactly the same as before.[6]

Now, we do not want to discuss here the sensibleness of the strategy of issuing debt (*i.e.*: borrowing money) and immediately buying back issued debt, but rather we want to stress the fact that if, for whatsoever reason, the borrower has money to reduce its outstanding debt, he/she is correspondingly cancelling also a part of the **DVA** shown in the balance sheet. In other words, the present value of the costs due to the funding spread can be reduced when the borrower has available free cash to buy back his/her own debts. If available cash is obtained by a new loan, no real funding benefit can be achieved; this is true also if the cash is originated by a derivative transaction (*e.g.*: selling an option), as we will see below.

The argument of Morini and Prampolini [10] does not seem to offer a true justification of a consistent insertion of the **DVA** as a reduction of the liabilities' value, even if one looks at it as a funding benefit and not under the perspective of counterparty credit risk, which we already criticized above. Actually also in the reasoning presented above we are not referring at all to counterparty credit risk, but we are simply referring to costs, provided that axiom 2.1 holds.

It is worth noticing that our notion of **DVA** does not exclude the possibility that the borrower may enjoy a reduction of his/her liabilities' value: if the interest rate rises, the present value of the loan decreases. If this profit can be actually realized depends on the composition of the assets of the borrower, since free cash available is needed to buy back the loan.

5 The DVA for Derivative Contracts

The **CVA** and the **DVA** are concepts devised for OTC derivative contracts as measures of counterparty credit risk. As such, they are improperly used for loan contracts, but ultimately their application offers a good conceptual framework to decide how to properly include the credit risk in the balance sheet also of a debtor and to precisely disentangle the contribution to the total P&L of the several cost and income components.

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[6]It is worth noticing here that we are simply confirming the ancient philosophical statement "*ex nihilo nihil*" (Melissus of Samos, 5^{th} c. BC): it is impossible to create something from nothing, or to get "blood from stones", as a more common-sense saying goes. On the other hand, the *funding benefit* argument, in our opinion, is in striking conflict with another important principle of the Aristotelic logic: "**A** cannot be **A** and at the same time **not A**", the $\pi\alpha\sigma\tilde{\omega}\nu\ \beta\epsilon\beta\alpha\iota o\tau\acute{\alpha}\tau\eta\ \alpha\rho\chi\acute{\eta}$, the firmest of all principles (*Metaph.* 1005b 15-17). In the specific case, we cannot say that borrowing money, for the borrower, is at the same time debt and not debt, as it seems if we allow for the existence of a funding benefit when debt is employed to replace a debt.

When dealing with OTC derivatives contracts, the main difference is that the exposure that one or both counterparties have to the other party is stochastic over time. We will investigate how the **DVA** can be included in the balance sheet and which is its interpretation in this case.

We analyse a very simple derivative contract: a forward contract on an asset S. The main set-up is the same as above: we assume that the bank (which is now no more a borrower) B strikes a deal in 0 to buy in T one unit of the asset S. We also assume, to simplify things, but with no loss of generality, that the counterparty of the bank is risk-free, so that we do not have to consider any **CVA** into the analysis. The default of the bank can occur only at the end of the activities in T.

The value of the contract $H(0)$ can be derived according to standard techniques[7] as:

$$H(0) = e^{-rT}(\mathbb{E}[S_T] - K) + \mathbf{DVA}_B(0) \tag{5}$$

where $\mathbb{E}[]$ is the expectation operator and the $\mathbf{DVA}_B(0)$ is (assuming independence between default probability and asset's price and zero recovery on default):

$$\mathbf{DVA}_B(0) = e^{-rT}\mathbb{E}[\min(H_T, 0)(1-\mathbf{1}_{\tau_B>T})] = e^{-rT}\mathbb{E}[\max(K-S_T, 0)](1-e^{-s_B T}) \tag{6}$$

In a verbose way, **DVA** is the discounted expected negative value of the contract at the expiry, weighted by the probability of default of the bank. This is the loss that the counterparty may expect to suffer, given the default of the bank. The fair forward price is the level of K making nil the value of the contract at inception:

$$H(0) = e^{-rT}(\mathbb{E}[S_T] - K) + \mathbf{DVA}_B(0) = 0$$

so that:

$$K = \mathbb{E}[S_T] + e^{rT}\mathbf{DVA}_B(0)$$

It is manifest that the bank can close a forward contract at conditions worse than those achievable if it were risk-free. In fact, B buys at the expiry the underlying asset at a price $K > K^{rf}$, where $K^{rf} = \mathbb{E}[S_T]$ is the fair forward price if B cannot go bankrupt, and hence the $\mathbf{DVA}_B(0)$ is zero.

Let us consider how to include in the balance sheet the forward contract (the equity is the same as in the case we have examined above). The value of the contract has to be computed by discounting the expected terminal value of the contract, plus the expected losses due to counterparty risks (*i.e.*: the **CVA**, which is nil in our case by assumption) and the other costs (the **DVA** in the framework we have suggested). The value of the contract to B is:

$$H_B(0) = e^{-rT}(\mathbb{E}[S_T] - K) = -\mathbf{DVA}_B(0)$$

which is negative and it is a (positive) liability (although it should be noticed that the value may change sign until maturity, and hence become an asset). The bank

[7] See Brigo and Capponi [3]. Although never specified until this point, it is now important to stress the fact that we are valuing all contracts under a risk-neutral measure, and all expectations should be considered as computed with respect to this measure.

has to recognize a liability due to the mark-to-market immediately after closing the deal, and this is equal to the **DVA** of the contract, as just shown above. On the other hand it has also to consider the **DVA** as the present value of the costs due to the fact that it is not default risk-free. So the balance sheet reads as:

Assets	**Liabilities**
$D_1 = E$	$-H_B(0) = \mathbf{DVA}_B(0) = e^{-rT}\mathbb{E}[\max(K - S_T, 0)](1 - e^{-s_B T})$
	E
	$-\mathbf{DVA}_B(0) = -e^{-rT}\mathbb{E}[\max(K - S_T, 0)](1 - e^{-s_B T})$

Assets and liabilities are manifestly balancing and we are consistently considering the value of the contract and the related extra costs born by B. In this way the closing of the deal generates no further P&L.

We have now to check what happens at time T. Assume that the underlying asset price S_T is equal to the expected price at the contract's inception $S_T = \mathbb{E}[S_T]$: the bank is then suffering a loss calculated from the value of the forward contract as follows:

$$H_B(T) = \ell = \mathbb{E}[S_T] - K = -\mathbf{DVA}_B(0)e^{rT}$$

The balance sheet in T is then:

Assets	**Liabilities**
$D_1 = Ee^{rT}$	$-H_B(T) = \mathbf{DVA}_B(0)e^{rT}$
	E
	$II_1 = E(e^{rT} - 1)$
	$-\ell = -\mathbf{DVA}_B(0)e^{rT}$

The loss has to be financed by the cash available in the bank account, where the original equity was deposited, so that the final form of the balance sheet in T is:

Assets	**Liabilities**
$D_1 = Ee^{rT} - \mathbf{DVA}_B(0)e^{rT}$	
	E
	$II_1 = E(e^{rT} - 1)$
	$-\ell = -\mathbf{DVA}_B(0)e^{rT}$

This confirms the fact that the **DVA** also for a derivative contract is the present value of a cost. Anyway, the definition can be slightly refined by moving one step forward. In fact, let us assume that the underlying asset's price at the expiry T is some $S_T \neq \mathbb{E}[S_T]$: the value of the forward contract is then:

$$H_B(T) = S_T - K = S_T - K^{rf} - \mathbf{DVA}_B(0)e^{rT}$$

which may result in a profit or a loss, depending on the level S_T. Anyway, when this value is compared with the corresponding value of a forward contract whose fair price was determined by assuming that B is a risk-free counterparty, it is straightforward to see that:

$$H_B(T) - H^{rf}(T) = S_T - K^{rf} - \mathbf{DVA}_B(0)e^{rT} - S_T - K^{rf} = -\mathbf{DVA}_B(0)e^{rT}$$

So the **DVA** is the cost that worsens the losses, or abate the profits, at the expiry T with respect to the same contract dealt by a risk-free counterparty: this cost, is once again, due to the fact that the bank is not a risk-free economic agent. If we introduce a multi-period setting we will have the same conclusion as above,[8] that is: the variability of the **DVA** allows to allocate portions of the total costs on the different sub-periods, but it is immaterial to determining the total cost, which is still the **DVA** calculated at the start of the contract.

It is worth also analysing what happens with derivatives starting with non-zero value at inception, such as options. Burgard and Kjaer [4] recently provided a proof on how to replicate a derivative contract including **CVA**, **DVA** and funding. Their approach relies on trading in counterparty's bonds to replicate the **CVA** and own bonds to replicate the **DVA**: the argument is very similar to that one in Morini and Prampolini [10] and it hinges on the funding benefit the one can achieve by buying back his/her own bonds. The existence of issued bonds to be bought back is assumed, otherwise the replica would not be possible: if we accept this assumption, the **DVA** inclusion in the balance sheet, as correction of the contract's value, would be fully justified because it can actually be replicated. Let us check if this is true.

At time 0, let the bank B sell a call option O expiring in T to a counterparty. The value of this contract is its risk-free value minus the **DVA**, with no **CVA** since the bank has no exposure to the counterparty.[9] The value can be written as (with the same assumptions made for the forward contract):

$$O(0) = e^{-rT}\mathbb{E}[S_T - K]^+ - \mathbf{DVA}_B(0) \tag{7}$$

where the $\mathbf{DVA}_B(0)$ is:

$$\mathbf{DVA}_B(0) = e^{-rT}\mathbb{E}[S_T - K]^+(1 - e^{-s_B T}) \tag{8}$$

We insert this contract in the balance sheet, where also a debt is present. The value of the debt is is equal to the value of the option and it is deposited in a risk-free bank account D_2. The value of the option contract to the borrower is the risk-free premium $V(0) = O(0) + \mathbf{DVA}_B(0) = O^{rf}(0)$, and the $\mathbf{DVA}_B(0)$ is accounted for, according to our proposed notion, as a loss:

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[8]The argument shown above has to be generalized by considering the period loss as $-\ell(0,T) = -(P_0 e^{rT}(e^{-s_B T} - e^{-s_B 2T}) + (P_T - P_0)e^{rT}(1 - e^{-s_B T})$, where $P_i = \mathbb{E}_i[\max(S_{2T} - K, 0)]$ and $\mathbb{E}_i$ is the expectation computed in i.

[9]The **DVA** has negative sign in this case since the sign of the contract is negative.

Assets	Liabilities
$D_1 = E$	$L = Ke^{-rT}$
$D_2 = Ke^{-(r+s_B)T} = O(0)$	$V(0) = O(0) + \mathbf{DVA}_B(0)$
$D_3 = O(0)$	
	E
	$-\mathbf{DVA}_B^T(0) = -e^{-rT}K(1-e^{-s_BT})$
	$-e^{-rT}\mathbb{E}[S_T-K]^+(1-e^{-s_BT})$

where $\mathbf{DVA}_B^T(0)$ is the total **DVA** including the option's and the debt's ones. Now, according to Burgard and Kjaer [4] the replica generates enough cash to buy back the debt. In fact in our example, we have cash deposited in the account D_3 equal to the premium received.[10] This can be used to buy back the outstanding debt, whose value is equal to the premium, as assumed above to make things as simple as possible. So the balance sheet now reads:

Assets	Liabilities
$D_1 = E$	$L = 0$
$D_2 = Ke^{-(r+s_B)T} = O(0)$	$V(0) = O(0) + \mathbf{DVA}_B(0)$
	E
	$-\mathbf{DVA}_B^T(0) = -e^{-rT}\mathbb{E}[S_T-K]^+(1-e^{-s_BT})$

The debt is now nil and the **DVA** has been updated. The funding benefit has to be verified at the expiry of the option, when the option is worth $O(T)$ and its $\mathbf{DVA}_B(T) = 0$. The P&L generated by the option is $-(O(T) - O(0)e^{rT})$.

Assets	Liabilities
$D_1 = Ee^{rT}$	$L = 0$
$D_2 = Ke^{-s_BT} = O(0)e^{rT}$	
$D_3 = -O(T)$	
	E
	$II_1 = E(e^{rT}-1)$
	$\mathbf{P\&L}(T) = -(O(T) - O(0)e^{rT})$

Apparently we do not have any loss deriving from the **DVA**, but this is a false perception. Actually, if the debit value adjustment is the extra-cost the bank has to pay for not being risk-free, then if we compare the final **P&L** with respect to the **P&L** of a risk-free bank we get:

$$(O(T) - O(0)e^{rT}) - (O(T) - O(0)e^{rT} + \mathbf{DVA}_B(0)e^{rT}) = -\mathbf{DVA}_B(0)e^{rT} = -\ell$$

[10] To keep things stuck to the heart of the matter, we are not considering the entire replication portfolio, but we are limiting the analysis to the hedging of the **DVA**, with the strategy suggested in Burgard and Kjaer [4].

So the **P&L** has an hidden cost that is equal to the $\mathbf{DVA}_B(0)e^{rT}$, but this is equal to the compounded **DVA** on the outstanding debt before it was bought back. So, given the funds available to the bank over the period, which are equal to $O(0)$, the loss incurred are in any case $\mathbf{DVA}_B(0)$, explicitly or implicitly shown in the balance sheet. In the end, also the argument by Burgard and Kjaer [4] does not justify the inclusion of the **DVA** in the balance sheet as a liabilities' reduction, as expected after having criticized the same argument by Morini and Prampolini [10].[11]

We would like to stress the fact that we are not saying that the replication strategy suggested by Burgard and Kjaer [4] is wrong because it is not possible to buy back issued bonds, or that the assumption of existing outstanding debt is weak (although no issued bonds available is something that may actually happen). We believe we have only proved that, however you define it, abating liabilities with the **DVA** is an accounting and financial mistake, indeed subtle but with huge practical impacts, whose effects can now be more consciously accepted or not by national and international regulators.

We can finally propose the following definition, which encompasses all the cases we have analysed so far:

Definition 5.1. *The debit value adjustment* **DVA** *is the compensation that a counterparty has to pay, when closing a contract, to the other party to remunerate the default risk that the latter bears and that is specularly measured as a credit value adjustment* **CVA**.

This compensation is the present value of the extra costs (given axiom 2.1), that the counterparty has to pay with respect to a risk-free counterparty and as such it ***must*** *be included in a marked-to-market balance sheet as a reduction of the equity.*

In a multi-period setting, the portion of the initial **DVA** *attributed at each period may depend on the stochasticity of the probability of default of the counterparty, and of the underlying asset of the contract, but in any case the total cost over the entire duration of the contract is still the* **DVA** *calculated at the beginning of the contract.*

6 Extension to Positive Recovery and Liquidity Risk

In the analysis above we have assumed that the loss given default of the exposure is full (*i.e.*: $\mathbf{Lgd} = 100\%$), and that the liquidity spread is nil ($\gamma = 0$). In this section

[11] An easier way to look at the **DVA** on an option would be to consider an immediate closing out of the short position by buying an identical option from a risk-free counterparty. In this case the premium paid by the bank would be $V(0) = O(0) + \mathbf{DVA}_B(0)$, and the loss, equal to $\mathbf{DVA}_B(0)$, would be financed with other available cash and written in the balance sheet as a reduction of the equity, thus confirming the new notion of **DVA** we have proposed.

At this point someone could rightly thing that the author of these pages would rather have written a simple three-line note and published it in a finance blog. Hopefully, the abundant and likely superfluous discussion contained in the present work will eventually give birth of a new way to look at the issue of a how to correctly strike a marked-to-the-market balance sheet.

we release these two assumptions and we verify what effects the relaxing produces.

Let us start with the case when $\mathbf{Lgd} < 100\%$ and $\gamma = 0$. It is very well known that the spread, in a reduced form setting to model the credit risk when the recovery is a fraction of the market value, is:

$$s = \pi \times \mathbf{Lgd} \tag{9}$$

This can be seen as an approximation of the formula for the loss given default on an exposure of amount K: $\mathbf{Lgd}_X K(1 - e^{-\pi T}) \approx \mathbf{Lgd}\pi T K \approx K(1 - e^{-sT})$, with $s = \pi\mathbf{Lgd}$. When valuing the expected value received at the expiry T, one gets:

$$Ke^{-rT} - Ke^{-rT}(1 - e^{-\pi \mathbf{Lgd} T}) = Ke^{-(r+s)T}$$

thus confirming equation (9). Given the market spread s and assuming a loss given default $\mathbf{Lgd}$, we can derive the probability of default trivially as: $\pi = s/\mathbf{Lgd}$.

With this information at hand, it is quite straightforward to adapt the framework above to the case when $\mathbf{Lgd} < 100\%$. Actually, the credit value adjustment (equal to debit value adjustment seen from the borrower side) in formula (2) can be written as $\mathbf{CVA}_B = \mathbf{DVA}_B = \mathbf{Lgd}_B K\mathbb{E}[1 - \mathbf{1}_{\tau_B > T}] \approx K(1 - e^{-s_B T})$. On the other hand the funding cost $\mathbf{FC}_L$ is computed with s_L, which now is equal to $\pi_L \mathbf{Lgd}_L$ instead of simply π_L, but this change will not affect the subsequent analyis at any rate, so that it continues identical as above for all the rest.

We add now a liquidity premium $\gamma \neq 0$. When included in the lender's spread, we have that $s_L = \pi_L \mathbf{Lgd}_L + \gamma_L$, and this is the new spread to insert in the quantity $\mathbf{FC}_L$ of equation (2), an no other effects are produced.

For the borrower's spread the treatment of the $\mathbf{DVA}$ deserves more attention.[12] Let us define the spread including the liquidity premium as $s^*_B = \pi_B \mathbf{Lgd}_B + \gamma_B$ and the spread including just the credit component as $s_B = \pi_B \mathbf{Lgd}_B$. Now equation (3) has to be modified as follows:

$$V_B = P_B - e^{-rT}[K - P_B\big(e^{(r+s^*_B)T} - e^{rT}\big)] = -P_B e^{s^*_B T} + Ke^{-rT} = 0 \tag{10}$$

and we have

$$P_B = e^{-rT}K - \mathbf{DVA}_B - \mathbf{LPC}_B$$

where

$$\mathbf{DVA}_B = \mathbf{CVA}_B = e^{-rT}K(1 - e^{-s_B T})$$

is the debit value adjustment, and

$$\mathbf{LPC}_B = e^{-rT}K(e^{-s_B T} - e^{-s^*_B T})$$

[12] Incidentally we here notice that $\mathbf{FC}_L$ is the funding cost of L, and this is implying that also L is actually a borrower of someone else's money. So when it comes to the disentangling of the the total funding cost $\mathbf{FC}_L$ into the $\mathbf{DVA}$ and liquidity component, what we will show for the borrower B actually applies also to the lender L.

is the liquidity cost due to the liquidity premium γ_B. The quantity $\mathbf{LPC}_B$ is an extra cost in all respects equal to the **DVA** for the borrower and hence it has to be included in the balance sheet as a reduction of the net equity, similarly to the debit value adjustment:

Assets	Liabilities
$D_1 = E$	$L = Ke^{-rT}$
$D_2 = Ke^{-(r+s^*_B)T}$	
	E
	$-\mathbf{DVA}_B(0) = -Ke^{-rT}(1 - e^{-s_B T})$
	$-\mathbf{LPC}_B(0) = -e^{-rT}(e^{-s_B T} - e^{-s^*_B T})$

The analysis then can be easily extended to consider also the costs related to liquidity.

It is worth here stressing that the sum of the $\mathbf{DVA}_B$ and the liquidity costs $\mathbf{LPC}_B$ are just the total funding cost $\mathbf{FC}_B$ for the borrower. In fact, if the borrower is taking money from economic agents not paying any funding spread, such as investors, than it is easy to see that (from the definition of P_B):

$$\mathbf{FC}_B = e^{-rT} P_B(e^{(r+s_B)T} - e^{rT}) = e^{-rT} K(1 - e^{-s_B T}) = \mathbf{DVA}_B$$

If the borrower has to pay also the funding spread charged by a lender who has to fund the activity, such as bank, then one gets:

$$\mathbf{FC}_B = e^{-rT} P_B(e^{(r+s_L+s_B)T} - e^{(r+s_L)T}) = \mathbf{DVA}_B + \mathbf{IC}_B$$

where the $\mathbf{IC}_B$ is the intermediation cost that the borrower has to pay to the lender for not having direct access to the capital market, and it is defined as:

$$\mathbf{IC}_B = e^{-rT} P_B(e^{s_L T} - 1)(e^{(r+s_B)T} - e^{rT})$$

Although we left unspecified, the funding cost of the lender $\mathbf{FC}_L$ is actually the sum of his/her $\mathbf{DVA}_L$ (and intermediation costs $\mathbf{IC}_L$ in case) and liquidity costs $\mathbf{LPC}_L$.

We are now able to give a definition of the funding cost for loan and for derivative contracts:

Definition 6.1. *The funding cost* **FC** *for a loan contract is the present value of the extra-costs, with respect to a risk-free counterparty, that a counterpary has to pay for the liquidity premium, for the intermediation costs for not having direct access to the capital market, and to compensate the other party for the default risk that the latter bears.*

The funding cost **FC** *related to a derivative contract is the the sum of the funding costs that a counterpaty has to pay on money it borrows to match negative cash flows generated by it. Given that the present (risk-free discounted) value of the sum of (expected) negative and positive cash-flows is nil when the contract is fairly priced, the borrowing of money is needed only when cumulated cash-flows are negative during the life of the contract, before receiving counterbalancing flows. So, funding costs for a derivative contract depend on the cash-flow schedule and they may materialize or not.*

From the definition above it can be deduced that, assuming no liquidity premiums and intermediation costs, the funding cost and the debit value adjustment are the same thing for a loan contract. For derivative contracts the debit value adjustment is completely unrelated to the funding costs, which can be seen as the sum of the **DVA**s referring to the loans needed to fund negative cumulated cash-flows during the life of the contract. The evaluation of these costs has to be carried out on a case by case basis, depending on the type of contract and even on the side (long/short) that the counterparty is taking in it.

7 Recapitulation of Results

We recapitulate in the table below the main quantities we have studied above, their nature and the relationships exiting amongst them.

Quantity	Nature
CVA=-DVA	**Compensation** for the counterparty risk borne by a party, given the exposure, the probability of default and loss given default.
DVA	**Cost** paid by a party that worsen contract's conditions with respect to a risk-free counterparty, given the exposure, the probability of default and loss given default.
IC	**Cost** paid by a party for not having direct access to the capital market.
LPC	**Cost** paid by a party for the premium required in the market. to provide liquidity.
FC= DVA+LPC+IC	**Cost** paid by a party over the risk-free rate to raise funds. Some components may be nil.

8 Accounting Standard and DVA.

International accounting standards (IAS and FAS) agree on the inclusion of the **DVA** into the fair value of the liabilities of a bank.[13] Alternatively said, the revaluation of liabilities taking into account the credit risk of the issuer (or of the counterparty with negative NPV in a derivative contract) is possible.

IASC [5], the board setting IAS accounting standard, tries and justify the inclusion of the **DVA** as a liability reduction:

> However, the Board noted that because financial statements are prepared on a going concern basis, credit risk affects the value at which liabilities could be repurchased or settled. Accordingly, the fair value of a financial liability reflects the credit risk relating to that liability. Therefore, it decided to include credit risk relating to a financial liability in the fair value measurement of that liability for the following reasons:
> (a) entities realise changes in fair value, including fair value attributable

[13]The expression "debit value adjustment", or its acronym **DVA** is never used in the documents of the accounting standards but, although with a different wording, its inclusion in the balance sheet as a reduction of the current value of the liabilities is clear.

> to the liability's credit risk, for example, by renegotiating or repurchasing liabilities or by using derivatives;
> (b) changes in credit risk affect the observed market price of a financial liability and hence its fair value;
> (c) it is difficult from a practical standpoint to exclude changes in credit risk from an observed market price; and
> (d) the fair value of a financial liability (ie the price of that liability in an exchange between a knowledgeable, willing buyer and a knowledgeable, willing seller) on initial recognition reflects its credit risk.
>
> The Board believes that it is inappropriate to include credit risk in the initial fair value measurement of financial liabilities, but not subsequently.

This is a basis for a conclusion (BC89) of the IAS 39 document: needless to say that the entire assertion is weak. The first part astoundingly cites the "going concern basis" to justify (also) the reduction of the liabilities due to the credit spread of the issuer: we think we have demonstrated at length why the inclusion should be as a reduction of the net equity, which entails also a more sensible effect when the credit standing changes.

Besides, point (a) is simply false as far as credit spreads are considered,[14] except in the part when saying "by repurchasing liabilities", but in this case is rather unlikely to have a widening of the credit spreads (and hence a reduction of the value in liabilities) and to be enough cash rich to buy back debt. Realizing the revaluation profits "by using derivatives" means for the bank selling CDS protection on its own debt, which is clearly not possible. Renegotiating debt means that that the bank is trying to update the interest it is paying on its debt, so as to get the new value of liabilities more in line with the notional value: under a financial point this new situation is exactly the same as the starting one.

Point (b) is a truism, not really supporting the general view. Point (c) is probably the most sensible statement, at least trying to find a practical reason instead of a convincing and sound justification. Point (d) is again a truism. The last statement is frankly not even worth a comment, given its complete lack of rational sense (better, lack of comprehension of financial contracts' valuation).

According to FASB [2], the board setting FAS standard:

> The reporting entity should consider the effect of its credit risk (credit standing) on the fair value of the liability in all periods in which the liability is measured at fair value because those who might hold the entity's obligations as assets would consider the effect of the entity's credit standing in determining the prices they would be willing to pay

[14] As noted above, a change in the market rates implying a reduction of the liabilities' value is admissible in our framework, and it could be even be monetized by a buy-back of issued debt. So, when the change in the current value of the liabilities is due to factors other than credit risk, it can be assumed that the revaluation profit/loss can be recognized in the balance sheet as a correction of the liabilities' fair value.

The revaluation of the liabilities including the entity's credit spread is then supported with claims such as:

> Like all measurements at fair value, fresh start measurement of liabilities can produce unfamiliar results when compared with reporting the liabilities on an amortized basis. A change in credit standing represents a change in the relative positions of the two classes of claimants (shareholders and creditors) to an entity's assets. If the credit standing diminishes, the fair value of creditors' claims diminishes. The amount of shareholders' residual claim to the entity's assets may appear to increase, but that increase probably is offset by losses that may have occasioned the decline in credit standing. Because shareholders usually cannot be called on to pay a corporation's liabilities, the amount of their residual claims approaches, and is limited by, zero. Thus, a change in the position of borrowers necessarily alters the position of shareholders, and vice versa.

Also in this case, although there is a tentative justification based on a micro-economic basis, the very slippery ground which it stands on is patently indicated by the wording "but that increase probably is offset by losses that may have occasioned the decline in credit standing". Beyond that, it is rather irrational to consider within the balance sheet, which represents the value of the company from shareholders' perspective, the value of the liabilities from the creditors' perspective. To the debtor, the value of the liability is just the present value (discounted at risk-free rate) of the notional amount (thus strictly adhering to the going concern principle). The "change in the position of borrowers necessarily alters the position of shareholders",[15] this is true, but the balance sheet should report just the latter position and not mix both together. And the correct representation of the bank's value to the shareholders is given if the **DVA** is considered as a cost (loss) abating the net equity.

In conclusion, we think that the current accounting standards are not very firmly grounded and they allow for accounting conducts by banks that may produce very misleading information to investors relying on balance sheets' data. Hopefully in this work we contributed to better consider how to properly mark-to-market liabilities and how to represent the costs related to the credit standing of the debtor (which are just costs and not gains, as they appear according to existing accounting standards and practices).

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[15]The FASB very likely meant "creditors" instead of "borrowers", otherwise the assertion is hardly understandable. Our interpretation should be confirmed by the fact the the paragraph is referring to the "two classes of claimants (shareholders and creditors)". The borrower is the bank, and it has no claim on its own liabilities.

9 Conclusion

In this work we hope we have contributed to the understanding of the nature of the debit value adjustment, naively considered until now as the credit value adjustment seen by the opposite side of the contract. We propose a new definition of **DVA** as present value of future costs; the new definition derives from a simple and sensible axiom, sometimes referred to as going-concern principle, and from a clear approach to pricing a deal by discounting all the expected cash-flows and costs.

The analysis has also shed some light on how to discount future cash-flows: the discounting should always be operated via the risk-free rate. Rates including credit and liquidity spreads are simply effective rates that allow to take into account in a synthetic fashion different risks into the pricing. If for pricing purposes this could be acceptable, under an accounting and risk management perspective it is more useful to keep all the expected cash-flows and costs separated and use a single discounting rate.

We are sure that the new perspective we offer will be useful to design more consistent trading and position keeping systems: these currently mainly rely on the idea to use different discounting curves to apply to different contracts and counterparties. We think that this approach is not satisfactory and it can be improved by shifting the complexity of the pricing to the disentangling of the different components of the cash-flows and related costs.

A problem still left open is how to apply the ideas we have shown to the market practice to sign CSA agreements to mitigate the counterparty risks. It has been shown in many works (see Bianchetti [1], Mercurio [9], Piterbarg [11]) that the discounting for a contract that is collateralized continuously should be operated by the collateral rate. The extension of the conceptual framework presented above to this case is left to future investigation.

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