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Network Analysis Applied to the Euro Stoxx 50: Impact of the ESG Score on the Stock Market

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Front Cover: **Roberto Crippa**, *Spirale*, 1971.

Executive Summary

With this work it has been attempted to understand if and how the ESG score of the companies plays a role in the creation of replicating portfolios of market indices, more precisely in the case of the Euro Stoxx 50.
Complex networks are built using the daily closing price time series of the fifty stocks that make up the Euro Stoxx 50 index in the time period between 3 September 2019 and 31 August 2021. These stocks, which correspond to quoted companies, are the nodes of the network while the links are determined thanks to a binary approach which involves the insertion of a connection between two nodes only if their cross correlation has a larger value than a certain fixed threshold. Since all the networks constructed in this way are scale-free for high values of the correlation, it is possible to deduce that the main market changes can be captured by a small group of stocks that show similar variation profiles over a given period. It is therefore reasonable to expect that when a new index is created by selecting only a few of the most correlated initial companies it will be possible to obtain a good approximation of the Euro Stoxx 50 index. By comparing these results with those obtained by also introducing the ESG score of each company in the selection criterion of the stocks used to compose the new index, it can be finally concluded that there is a general improvement in the approximation of the evolution of the reference index. Moreover, focusing on the ESG score of the companies selected for the replication of the index in the two situations, it can be deduced that it is not necessary to have a high scoring to be an important player in the financial market.

About the Author

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Martina Cattaneo

Nowadays there are still no certain results about the impact of sustainable finance on financial markets. It is certainly not enough to invest in a more eco-friendly company just for ideological reasons; it is important to try to understand what this entails from a financial point of view and to have an idea of the riskiness and the return of an investment of this type.

Therefore, in this work a first analysis has been carried out, based on network techniques, in order to understand if the ESG score could affect the performance of a group of companies by capturing significant interconnections between them, that otherwise would not be pointed out, so as to highlight a real impact of the ESG score in the study of the financial market which allows to describe it more efficiently.

Moreover, generally the companies with higher ESG score obtain greater returns but this does not mean that they better describe the trend of the financial market. In fact, the other aim of this paper is to show that the influence of a corporation on the market does not depend only on its earnings but also on the interconnections that bind it to all the other players.

After an initial brief smattering on the Euro Stoxx 50 in Chapter 1 and on ESG score in Chapter 2, the core of the paper is the numerical analysis proposed in Chapter 3, in which a real dataset is analyzed to try to understand the impact of the ESG score in the calculation of a financial index.

First of all, have been constructed some networks composed of fifty nodes, each corresponding to one of the stocks that make up the Euro Stoxx 50 index on 3 September 2019, that have been linked only if characterized by a cross correlation bigger than a precise threshold value ρ. Then, thanks to some network theory tools, only three of these networks have been selected in order to proceed with the creation of a new index capable of replicating the trend of the Euro Stoxx 50 by making use of fewer companies selecting those that have a price more similar to each other. Subsequently, the ESG score has been introduced in the analysis so that the companies which make up the new index are chosen not only for the most similar price but also for the most common ESG score. At this point some interesting comparisons between the networks constructed with the same values of ρ in the two different situations have been made, more precisely are compared the first case in which the ESG score is not present and the one that takes it into account in the selection criterion, in order to see whether the addition of the ESG score has led to real changes in the structure and in the composition of the networks. Finally, by comparing the obtained results regarding the price trends of the new index with respect to the reference one, it is possible to understand whether the introduction of the ESG score has been able to capture new similarities between companies, giving rise to a better approximation of the Euro Stoxx 50 price.

1. Euro Stoxx 50

The Euro Stoxx 50 is a stock index made up of fifty of the largest and most liquid stocks of the Eurozone but only companies located in countries where the Euro is in effect can be included in the index, whereby British corporations are not inserted in it [11].

It was introduced on 26 February 1998 by Stoxx Limited, an index provider owned by Deutsche Börse Group, that reviews annually, in September, its composition. It is available in several variant combinations (Price, Net Return, Gross Return) and its calculation depends on its currency (EUR, USD, CAD, GBP, JPY): it takes place every 15 seconds between 09:00 CET and 17:30 CET for the EUR and USD variants of any return type, while the CAD, GBP and JPY ones are available as end-of-day calculation only (17:30 CET). The all-time high value reached by the index was 5,464.43 points on 6 March 2000 while its minimum was touched on 5 October 1992 with a close at 920.65 points.

In practice to determine the fifty companies of the index, a ranking of the 600 corporations with the largest market capitalizations, called blue-chip, in the Eurozone is done: a new company must be in the top 40 to become part of the index while the corporations that are already in it must be in the top 60 to stay there. Finally, the Euro Stoxx 50 price is computed with the Laspeyres formula (detailed in Section 3.3) in which each company of the index has a weight depending on its market capitalization so that the corporations with a high market capitalization value have a greater relevance. It is important to notice that this calculation excludes dividends recorded by stocks.

The Euro Stoxx 50 is used by financial institutions as underlying for many investment products (*e.g.* Exchange Traded Funds (ETFs), futures, options and structured products) as well as by fund managers in order to measure their own performance [9].

2. ESG Score

As reported in an article on the Borsa Italiana website [2], a study done by Morningstar on March 2021 on 11,500 Luxembourg funds and ETFs (Exchange Traded Funds) and 30 asset managers, reveals that funds and ETFs that are defined as sustainable according to the SFDR (Sustainable Finance Disclosure Regulation) are at least 21% of the total, for a value of approximately € 2.5 trillion. According to this regulation, since 10 March 2021 the financial operators involved in the European Union markets must disclose information about the sustainability of their investment policies and products. The main purpose of this legislation is to increase the circulation of accurate and reliable information on the sustainability characteristics of investment policies and financial products, in order to encourage greater investments in sustainable projects and businesses thanks to a better understanding of the ESG risks. Clearly the full application of this regulation will take time because several common rules still need to be clarified and the companies have to start adapting to the new requirements by having to communicate many very precise data on how they take environmental, social and governance factors into account.

The ESG score is a synthetic evaluation of a corporation about its degree of sustainability in the environmental, social and governance pillars with the aim to measure the resilience of a company to long-term financially relevant risks. More precisely,

- **Environmental**: it includes everything that concerns the impact that companies have on the environment and on the territory. In practice it is related to the climate changes (*e.g.* interested in containing CO_2 emissions) and to the efficiency in the use of natural resources (*e.g.* water).
- **Social**: it takes into account the human capital in all its forms. The main aspects analyzed are the quality of the work place and the respect for human rights (*e.g.* gender diversity).
- **Governace**: it concerns the ethics and transparency of the corporate governance. The focus is on the diversity policies (*e.g.* gender and ethnic) in the composition of the board and on the presence of sustainability plans and objectives linked to its remuneration.

The ESG rating companies (*e.g.* MSCI, Bloomberg, Thomson Reuters and Morningstar) rank the corporations by a wide range of ESG issues based on a relative evaluation divided into different categories, which are usually inspired by the seventeen thematics that make up the United Nations Sustainable Development Goals of 2015, that are broken down into a set of key issues. In practice, they collect data from multiple sources, internal and external to each company analyzed, and they process them based on different metrics using personal models in order to aggregate, weight and finally get a final score.

For example MSCI, one of the main ESG rating providers, uses a quantitative and data-based method taking available indicators from reputable external sources. In order to better understand the correct management of a risk factor by a company, the MSCI scoring method is evaluative, based on scoring scales along the two dimensions of risk and opportunity analysis related to the most critical and/or relevant topics in terms of corporate social responsibility. In other words, the assessment is carried out both through exposure metrics, which measure how much the company is exposed to the examined risk, and through management metrics, used to quantify the effectiveness of the effort made by the corporation to deal with the risk itself. In this way, on the one hand the negative externalities produced by companies in a given sector are identified, which will turn into an unexpected cost in the medium-long term, and on the other, the opportunities that can be capitalised, again from a long term perspective. Once the collected information is converted into a score from 0 to 10, where a lower value indicates a more severe risk, all the results are combined together into a single number which is transformed into a seven-point scale where the 'AAA' rating represents the best-in-class ESG performance (*i.e.*, lower ESG risk exposure and stronger risk management) whereas the 'CCC' rating reflects the worst-in-class ESG performance [5].

Since each ESG rating company applies its own approach, disagreement results are sometimes obtained. The main discrepancy is based on the fact that the interpretation of the data is a qualitative practice and so each rating agency is free to consider in each pillar the dataset that it deems most appropriate. Also consider how much importance to assign to an indicator in the model is a largely subjective decision and it can happen that different agencies do not consider the same pillar as the more relevant one [3].

In recent years, actions and products in favour of the ESG parameters are increasingly appreciated by both investors and consumers, who pay more and more attention to topics of this kind. A high ESG score reflects the fact that the company is well disposed towards these issues and that it is able to best manage its exposures deriving from ESG risks and opportunities, helping to improve the brand image and to attract investments thanks to which it can reduce financing costs. But the transition processes that have to be adopted by corporations are not without risk: the new strategies can be mismanaged leading to a loss of customers due to a drastic departure from the typical principles of the company itself [8].

3. Numerical Analysis and Results

In this chapter a real dataset is analyzed thanks to the use of some network techniques. The starting point for this applicative part was Tse, Liu and Lau's paper [13] in which they introduced an alternative method to select only some stocks to be included in the calculation of some of the most famous American indices. Inspired by this work, the first purpose of this project consists in reducing the number of stocks that make up the Euro Stoxx 50 index in order to see if, in this way, there is an excessive loss of information or if the index computed with fewer companies still works well. Afterwards, inserting an actuality topic such as the ESG score, the whole analysis is repeated changing the way in which two corporations are considered similar in order to deduce if the ESG score could help to better describe the index performance and whether the companies with higher ESG ratings are major players in the financial market.

3.1 Dataset

The analysis is based on a two-year period, from 3 September 2019 to 31 August 2021, in order to consider a particular era in which the Euro Stoxx 50 was first affected by the collapse of the arrival of the Covid-19 pandemic and then gradually recovered and because at least a length of two years is usually chosen in order to capture similar trends between different variables.

In order to reproduce the Euro Stoxx 50 daily closing price using fewer companies, first of all it is important to search data for the fifty corporations that make up this index. To simplify the analysis, the composition of the Euro Stoxx 50 has been fixed at the initial date of the observations, on 3 September 2019, denominated t_0 even if it is well known that its composition changes over time. Therefore, knowing the actual composition of the index and searching all its variations during the considered period, the fifty European companies present at t_0 have been selected. In Table 1 are reported all the considered corporations together with the country where they are headquartered and the macro-sector in which they operate.

Summing up, as represented in the pie charts in Figure 1, the Euro Stoxx 50 evaluated at t_0 is made up of fifty stocks from nine different Eurozone countries: France (18 companies), Germany

Identifier (RIC)	Country	Macro-sector	Identifier (RIC)	Country	Macro-sector
LVMH.PA	France	C	ADSGn.DE	Germany	C
ASML.AS	the Netherlands	C	SAN.MC	Spain	K
OREP.PA	France	C	BMWG.DE	Germany	C
SAPG.DE	Germany	J	SGEF.PA	France	F
LINI.DE	England	G	SAF.PA	France	C
SIEGn.DE	Germany	M	ISP.MI	Italy	K
VOWG_p.DE	Germany	C	INGA.AS	the Netherlands	K
TTEF.PA	France	C	BAYGn.DE	Germany	G
SASY.PA	France	G	DANO.PA	France	G
ITX.MC	Spain	G	ENI.MI	Italy	C
AIR.PA	the Netherlands	C	BBVA.MC	Spain	K
SCHN.PA	France	C	PHG.AS	the Netherlands	M
ABI.BR	Belgium	I	MUVGn.DE	Germany	K
DTEGn.DE	Germany	J	CRH.I	Ireland	G
DAIGn.DE	Germany	C	AD.AS	the Netherlands	G
ALVG.DE	Germany	K	VIV.PA	France	J
PRTP.PA	France	G	AMA.MC	Spain	J
ESLX.PA	France	C	ENGIE.PA	France	D
ENEI.MI	Italy	D	NOKIA.HE	Finland	J
DPWGn.DE	Germany	H	FREG.DE	Germany	Q
BNPP.PA	France	K	ORAN.PA	France	J
AIRP.PA	France	G	SOGN.PA	France	K
BASFn.DE	Germany	G	TEF.MC	Spain	J
IBE.MC	Spain	D	URW.AS	France	L
AXAF.PA	France	K	UL	England	C

TABLE 1: *Information of the companies that make up the Euro Stoxx 50 at time* t_0

(13), Spain (6), the Netherlands (5), Italy (3), England (2), Belgium (1), Ireland (1) and Finland (1). At a geographical level, therefore, French and German stocks are the ones with greater weight, while three countries have a unique company in the basket. It is very important to notice that, at the considered time t_0, in the index composition there are also two English companies because on 3 September 2019 the United Kingdom was still part of the European Union, from which it left on 31 January 2020 starting a transitional period that lasted until 31 December 2020.

Another interesting classification can be done dividing the companies according to the Ateco code (accessible on the *Istat website*) which allows to split the economic activities into 21 macro-sectors. In particular, the Euro Stoxx 50 covers eleven macro-sectors: C (14 companies), G (10), K (9), J (7), D (3), M (2), F (1), H (1), I (1), L (1) and Q (1). More precisely, Manufacturing Activities is the most popular because it comprehends a wide variety of industries, from auto to electrical components producer, passing through the apparel one. Other macro-sectors with many participants are Wholesale & Retail Trade and Financial & Insurance Activities: the first one contains, among others, pharmaceutical products and commodity chemicals. The last relevant macro-sector is the Information & Communication Services which includes, in addition to telecommunications, also the software sector. All the other macro-sectors are not interesting because they have a low number of companies within them, only one in five out of seven cases. Making a deeper analysis, it can be seen that the most popular sector is the Banks one, that is part of the macro-sector of Financial & Insurance Activities, which includes a total of six companies.

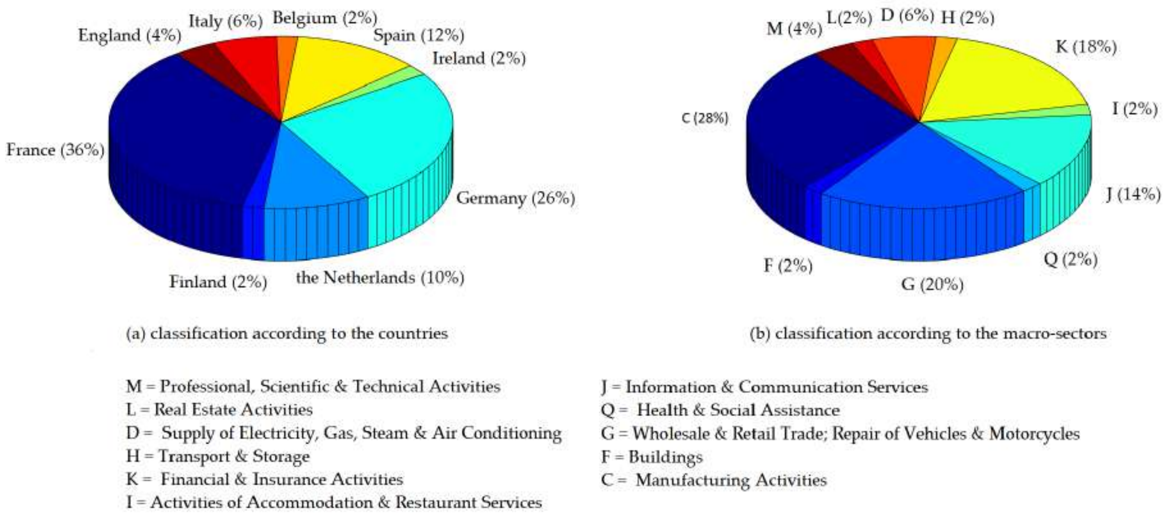

FIGURE 1: *Euro Stoxx 50 composition*

3.2 Network Construction

Once the time series of the daily closing prices for the fifty selected companies have been collected by *Thomson Reuters website*, the next step consists in constructing a network in order to apply network theory techniques to reduce the number of considered corporations. More precisely, each node of the network corresponds to one of the traded stocks, which are also the quoted companies, composing the Euro Stoxx 50 index at time t_0 while, as regards links, a link is inserted between a couple of nodes if the similarity between them, measured thanks to the Pearson correlation, exceeds a certain threshold value ρ. Since in this context it is significant to know just the amount of similarity between the stocks and not whether they are correlated in a positive or negative way, the rule used in order to construct the matrix correlation is:

$$|c_{ij}| > \rho, \tag{1}$$

where the Pearson correlation between the daily closing prices of stocks i and j, $p_i(t)$ and $p_j(t)$ respectively, is simply computed as:

$$c_{ij} = \frac{\sum_{t=1}^{N}[(p_i(t) - \bar{p}_i)(p_j(t) - \bar{p}_j)]}{\sqrt{\sum_{t=1}^{N}[(p_i(t) - \bar{p}_i)^2]}\sqrt{\sum_{t=1}^{N}[(p_j(t) - \bar{p}_j)^2]}}, \tag{2}$$

where $\bar{p}_i$ and $\bar{p}_j$ are the averages of the two considered time series. In this way the obtained network is undirected, since in the correlation there are not a source and a target node but both have the same role, and unweighted, because each link is not associated with a weight corresponding to the value of the correlation between the two connected nodes but all links have the same weight, to which value 1 is associated.

The correlation matrix, in absolute value to better capture the similarity between the different stocks, obtained with the collected dataset is shown in Figure 2.

As expected, the elements on the main diagonal are all equals to 1 while, regarding the others, in general there is a good correlation among the considered stocks since 52.65% of the correlation coefficients have a value above 0.5. In particular, it immediately catches the eye that SAPG.DE and SASY.PA are only correlated each other and just with another company, AD.AS and FREG.DE respectively, having the correlation values with all the other stocks very close to 0. Moreover, a particular high correlation, 0.9636 on average, can be noticed between LVMH.PA, ASML.AS & OREP.PA, all involved in the Manufacturing Activities, while a medium-high correlation is visible in four groups: LINI.DE, SIEGn.DE & VOWG_p.DE with an average of 0.8257; AXAF.PA, ADSGn.DE, SAN.MC, BMWG.DE, SGEF.PA, SAF.PA, ISP.MI & INGA.AS with a mean of 0.7941; BAYGn.DE, DANO.PA & ENI.MI with an average of 0.8146 and FREG.DE, ORAN.PA, SOGN.PA, TEF.MC & URW.AS with a mean of 0.7681. By carefully checking the values of the correlation matrix, only six stocks show all positive ones (ALVG.DE, ESLX.PA, BNPP.PA, ISP.MI, INGA.AS & MUVGn.DE)

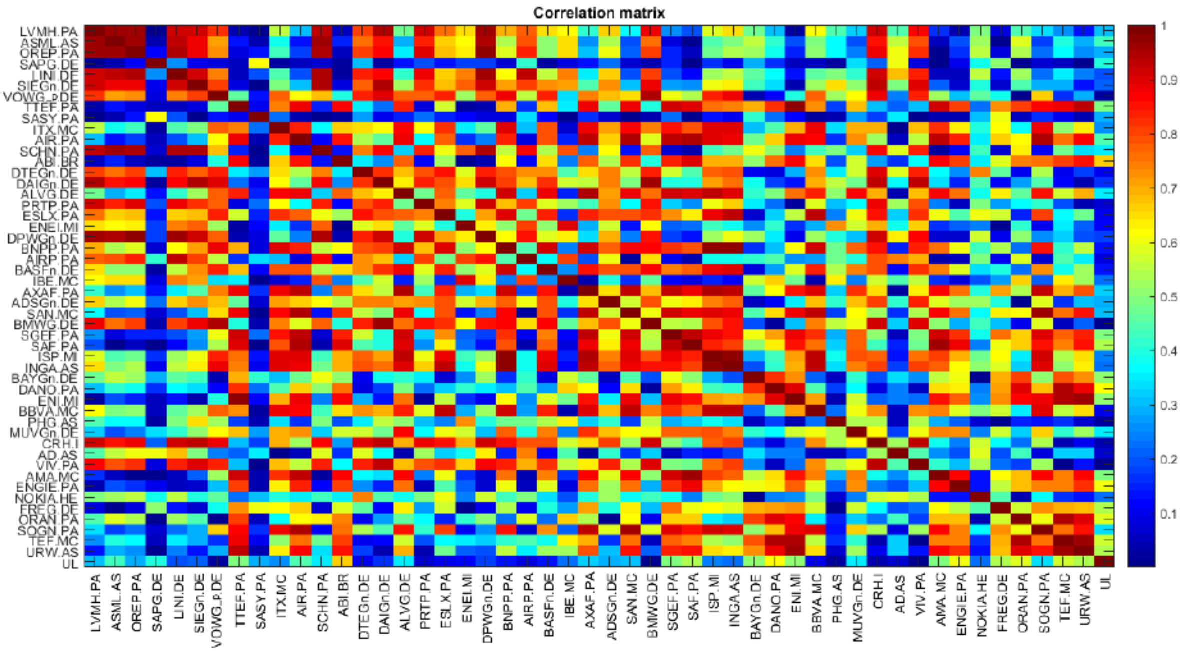

FIGURE 2: *Correlation matrix for the daily closing prices of selected stocks from 3 September 2019 to 31 August 2021*

while all the others have at least a negative value. Moreover, the stock with the largest number of negative correlation values is ORAN.PA, with 22 cases over 49.

Since the value of the threshold ρ is the key point for the creation of the network, in Figure 3 are illustrated six different graphs obtained increasing the value of ρ in order to get a general idea of how the choice of this parameter can affect the network itself. The values of the threshold have been chosen in the range in which the network has at least one link, starting from the extreme case of ρ = 0 which corresponds to a complete network.

By construction, as the threshold value increases there are fewer and fewer links in the network, until ρ becomes greater than 0.9751, value beyond which they all disappear. Moreover, it can be noticed that until more than the half value which can be assumed by the parameter ρ, the network is represented as a unique giant component. Instead, by increasing the value of the threshold, a lot of lonely stocks begin to appear and the network is divided in different components, each of which with fewer nodes: in the fourth case there are two components, in the fifth one the disconnected components grow becoming three but each one with no more than twelve stocks and, finally, in the last example only two stocks are connected through a link.

Thanks to these graphs it is immediately to see that, as just deduced above, SASY.PA and SAPG.DE are the less correlated stocks, while the link that disappears last is the one that connects INGA.AS and ISP.MI, both engaged in the banking sector. Furthermore, by better analyzing the different components formed in the case of ρ = 0.9353, it is possible to notice that the two biggest ones are mostly made up of companies operating in the same macro-sector: in the group located at the top left of the graph representation seven out of ten corporations are classified with C code, excluding SIEGn.DE, DPWGn.DE & LINI.DE, while the other one contains all the companies involved in the Financial & Insurance Activities macro-sector, only MUVGn.DE is excluded, as well as four others (AMA.MC, SAF.PA, AIR.PA & SGEF.PA). About the remaining component, involving DANO.PA, TEF.MC, ORAN.PA, URW.AS, ENI.MI & TTEF.PA, there are no basic relationships between the connected companies because they belong to different macro-sectors and they are situated in different countries.

In order to have a general picture of the just mentioned networks, a more detailed analysis has been made computing some interesting network parameters reported in Table 2: the number of nodes N, the amount of links L, the density d (measure of the interconnection between nodes), the average degree k (measure of the average number of neighbors of the nodes), the heterogeneity parameter h (sign of variability in the network based on the degree distribution), the diameter D, the average path length avg_l and the average clustering coefficient C (measure of the average

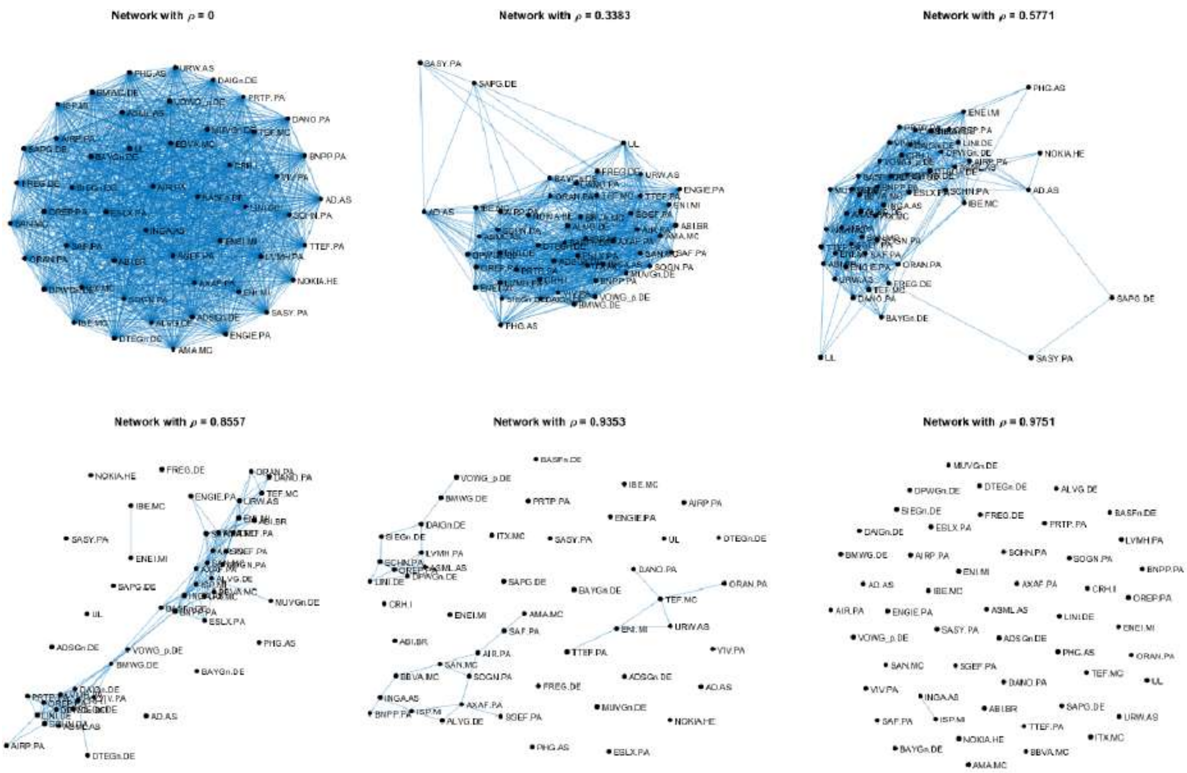

***FIGURE 3:** Examples of graph representation for six values of the threshold ρ*

connectivity of the graph thanks to the count of the triangles $\tau(i)$ involving the node i).

Clearly the number of nodes is always fifty because no company is eliminated changing the value of the threshold. Data in hand, it is possible to confirm what was said previously from the graphs: both the number of links and the density diminish for increasing values of ρ; they are already reduced to about 70% with a threshold value of only 0.3383. As a consequence, also the average degree is characterized by a degrowth, tending to zero when only a link remains in the whole network. As expected, the complete network results homogeneous, h = 1, because all nodes have the same number of neighbors and then, as ρ increases, the heterogeneity parameter takes on an ever greater value, a sign of a more variable degree distribution and therefore of the greater possibility of having hubs. Notice that, in general, all the considered networks have short paths, being the diameter not greater than 7 and the maximum average length near 2. Finally, regarding the average clustering coefficient, two results are exactly what is expected: for the complete network its value is 1, since all possible triangles are present, and for the last example it is not defined, because there are only two connected nodes and the requirement to compute the quantity is to have the degree bigger than one. About the intermediate cases, it is interesting to notice how this coefficient does not present a precise ascending or descending trend at the variation of the threshold parameter, a clear message that this quantity not only depends on the amount but also on the quality of the links.

3.3 New Index Construction and Discussion

Since the aim of this first part of the work is to try to recreate the daily closing prices of the Euro Stoxx 50 for the period from 3 September 2019 to 31 August 2021 using less corporations, it is important to make sure that this makes sense. For this reason, it is necessary to check if the created network is scale-free and so if there are few companies, called hubs, with a lot of connections and many corporations with few ones. In fact if this happens, since the network has been constructed based on similarities such that the hubs are the stocks more similar to all the other nodes, it makes sense to think that small variations on the daily closing prices of the hubs are able to affect the whole network and so that the index can be described only by these few stocks which capture the main characteristics of the network.

	$\rho = 0$	$\rho = 0.3383$	$\rho = 0.5771$	$\rho = 0.8557$	$\rho = 0.9353$	$\rho = 0.9751$
N	50	50	50	50	50	50
L	1225	820	539	162	38	1
d	1	0.6694	0.4400	0.1322	0.0310	8.1633×10^{-4}
k	49	32.80	21.56	6.48	1.52	0.04
h	1	1.0552	1.1430	1.5146	2.1814	25
D	1	3	4	7	6	1
avg_l	1	1.3322	1.6914	1.9575	2.1253	1
C	1	0.7761	0.7332	0.7394	0.4348	NaN

TABLE 2: *Network parameters for the six analyzed graphs*

From the results of the previous section, in particular from the heterogeneity parameter, it is possible to deduce that the network is scale-free for very high values of the threshold ρ. In order to find a more precise range of this parameter in which the network is scale-free, a visual approach based on its probability distribution has been adopted: since it is well known that a scale-free network has a power-law probability distribution, first of all it has been approximated by a decreasing exponential of the form:

$$P(k) = \alpha e^{-\gamma k}, \tag{3}$$

where the two parameters α and γ have been found applying the least square method. Then, the probability distribution and the fitted curve have been pictured in a log-log plot in order to better catch the scale-free property: in this plane, in fact, the power-law is represented as a decreasing straight line. Proceeding gradually narrowing the range of analysis, it has been found that the network results scale-free approximately in the interval of ρ [0.9308, 0.9636].

Once obtained the range in which the network is scale-free, the next step is to select a value of ρ in order to reduce the considered network and proceed with the analyzes. In reality, it has been decided to continue in two different ways in order to obtain more robust results.

The first approach simply consists in selecting the value of the threshold which gives the least mean fitting error, computed as the sum for each degree of the absolute value of the difference between the probability distribution and the approximated one:

$$\varepsilon_{fitting} = \sum_k |p(k) - \alpha e^{-\gamma k}|, \tag{4}$$

in the hope that this property will also be reflected in the reproduction of the daily closing prices. In this case the resulting value of the threshold is $\rho = 0.9428$.

About the other method, it is based on the links removal strategies [1]. In literature there are a lot of these procedures based on the different measures of nodes but in this work, according to the computed quantities, it has been decided to proceed with the so called weak strategy. In this technique, links are deleted in increasing order of weight so that, at the end, only the most strong connected ones will remain. In practice this is the strategy used from the beginning to construct the network because inserting a link only if the correlation is above a certain value is exactly the same thing. Once adopted a link removal strategy, it is needed to measure the network functioning in order to understand how it is affected by the deletion of some links because the aim is to make use of as few companies as possible without losing important information. For this purpose, two measures have been calculated: the Largest Connected Cluster (LCC) and the Efficiency (Eff).

The LCC is the giant component of the network so its measure is simply given by its number of nodes:

$$LCC = \max_j S_j, \tag{5}$$

where S_j is the size of the component j.

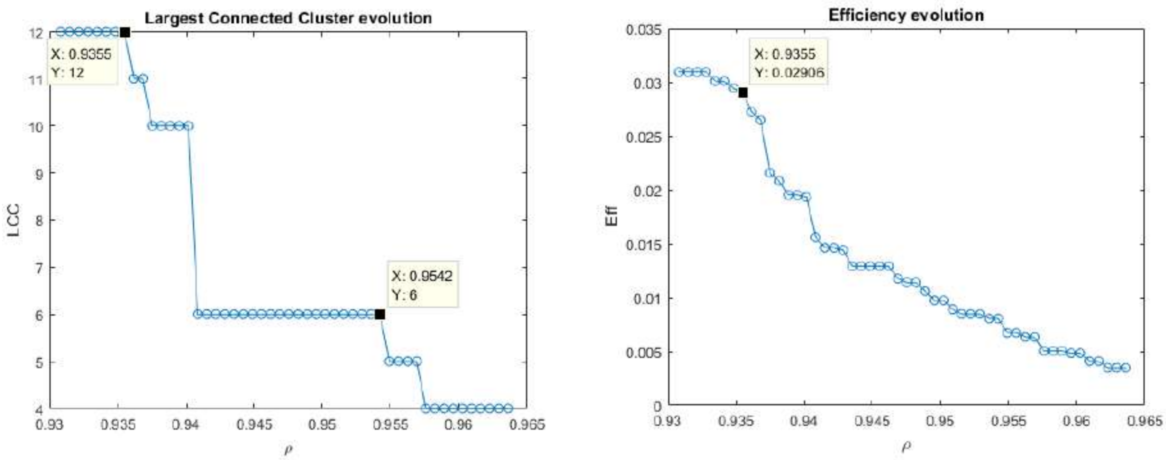

FIGURE 4: Plot of the LCC and the Eff measures in the scale-free range

Instead the Eff measures how good the information is exchanged within the network based on the idea that it is better spread if the distance is less so that:

$$Eff = \frac{1}{N(N-1)} \sum_{i \neq j \in G} \frac{1}{l_{ij}}, \tag{6}$$

where l_{ij} is the shortest path length between nodes i and j.

The evolution of these two measures in the scale-free range is plotted in Figure 4.

Obviously, both the quantities are decreasing functions because, augmenting the amount of removed links, the network becomes more divided and less efficient. By remembering that the purpose of this part of the work consists in reproducing the Euro Stoxx 50 daily closing prices as faithfully as possible also with a reduced number of stocks, the interesting points to be chosen are those positioned just before a step: here the network is still stable so it is possible to capture the moment just before the links removal changes it drastically, leading to an important loss of information. By looking at both the graphs, two interesting threshold have been taken: 0.9355 and 0.9542.

At this point, for each of the three chosen values of ρ, a subgraph has been constructed starting from the initial network and eliminating the isolated nodes obtaining the graphs illustrated in Figure 5.

In all the three cases, the new networks are divided into different components characterized by the fact that their size decreases as the ρ value augments: for ρ = 0.9355 there are three components with no less than six nodes each, about ρ = 0.9428 the network results more disconnected, because composed of six components, but the smallest size goes down to two and, finally, regarding ρ = 0.9542 the components are five and this time six is the largest size. It is interesting to notice that the smallest component of the first graph, that includes DANO.PA, TEF.MC, ORAN.PA, URW.AS, ENI.MI & TTEF.PA, is kept almost untouched in all the three cases: both stocks and links remain unchanged passing from ρ = 0.9355 to ρ = 0.9428 while, for the last step, two nodes are lost, DANO.PA and ORAN.PA, and the links are halved. Regarding the other groups, in the biggest one eight out of twelve companies are engaged in Financial & Insurance Activities and, with the increase of ρ, some of the new components present interesting characteristics: for ρ = 0.9428 the group composed of ALVG.DE, AXAF.PA, ISP.MI, INGA.AS & BNPP.PA includes only companies operating in the Financial & Insurance Activities and for ρ = 0.9542 it is reduced to contain only the three stocks operating in the Banks sector, as happens for the couple SOGN.PA & SAN.MC. About the last component of the first case, it is made up of seven out of ten corporations which come from the Manufacturing Activities macro-sector. With the growth of ρ it is broken into two groups, one which keeps the same six nodes (SCHN.PA, LINI.DE, OREP.PA, DPWGn.DE, LVMH.PA & ASML.AS) for both the remaining cases and the other one which is composed only of German stocks (SIEGn.DE, DAIGn.DE & BMWG.DE) that at the end are reduced to the couple DAIGn.DE & BMWG.DE, both operating in the Auto & Truck Manufacturers.

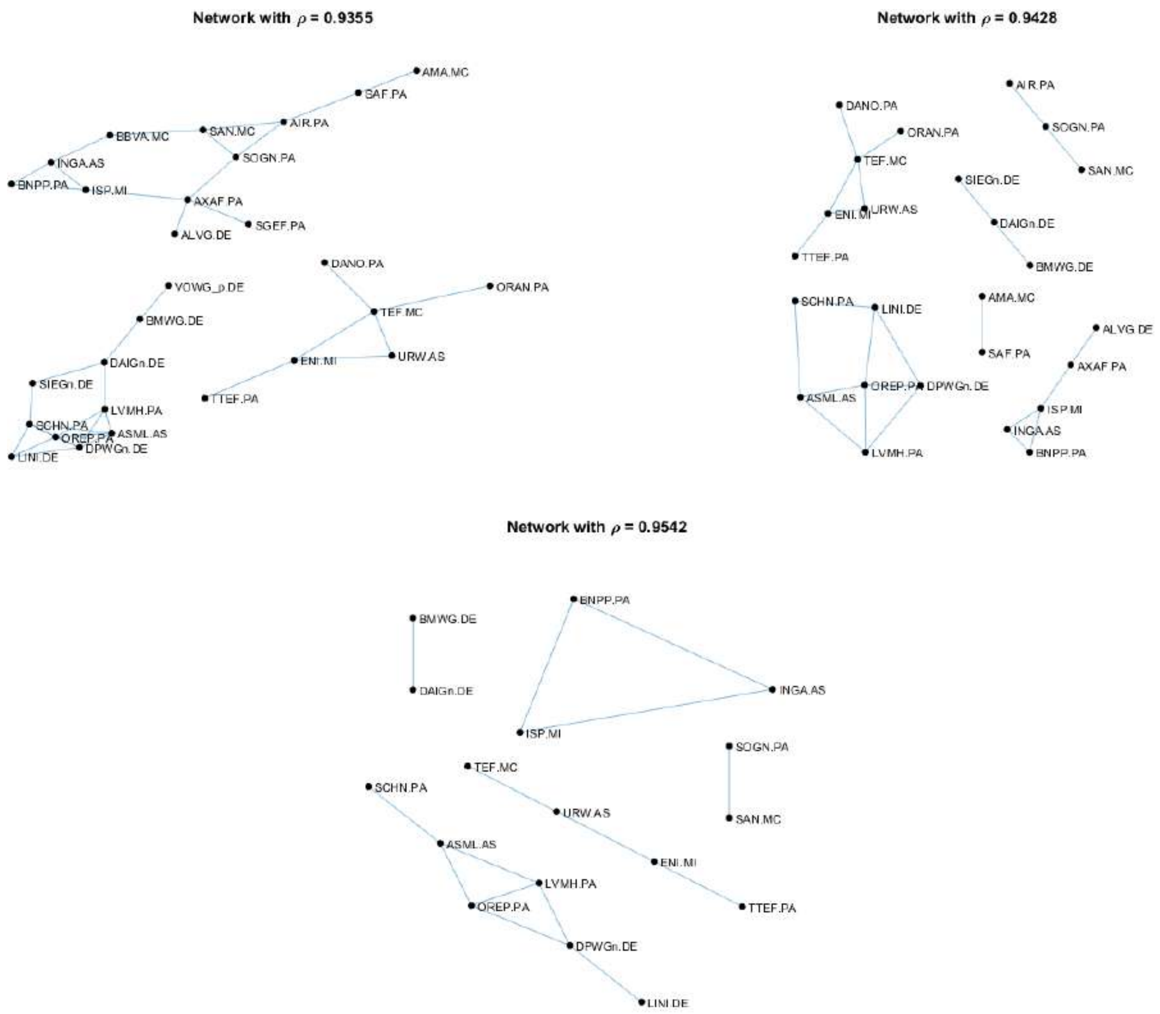

FIGURE 5: *Graph representation for the three new subnetworks*

If the probability distributions for these three networks are plotted, Figure 6, it is immediate to see that the central case is the best fitting since the red line passes very close to the circles corresponding to the values extrapolated from the dataset. About the other two examples, they both describe a scale-free network even if, in the first case, it is less evident. Finally, in Table 3 are reported the power-law parameters, α, γ and $\varepsilon_{fitting}$, for these three cases. As expected, the mean fitting error is smaller in the second case.

Now that for three values of the threshold only some stocks have been selected and a subnetwork has been constructed, a new index price has been computed in order to compare it with the Euro Stoxx 50 one and to see if, using only the most similar stocks of the index, much or little information is lost.

Usually the indices are calculated thanks to the Laspeyres formula [4] which measures price changes using the base period quantities. In its simplest formulation each company has a weight depending on its market capitalization and it is based on the assumptions that the components of the index and their number of shares remain the same for all the analyzed time period.

In addition there are other more sophisticated methods to calculate indices [10] among which a very interesting example is when only a part of the outstanding shares is considered [7], the free float which is the number of shares that is available to the public for trading in the secondary market, and an index divisor is inserted in the formulation in order to normalize it. In this work, as was done in the main reference article, a simple Laspeyres formula has been used where the new index at time t is simply computed as the market capitalization at the considered time normalized

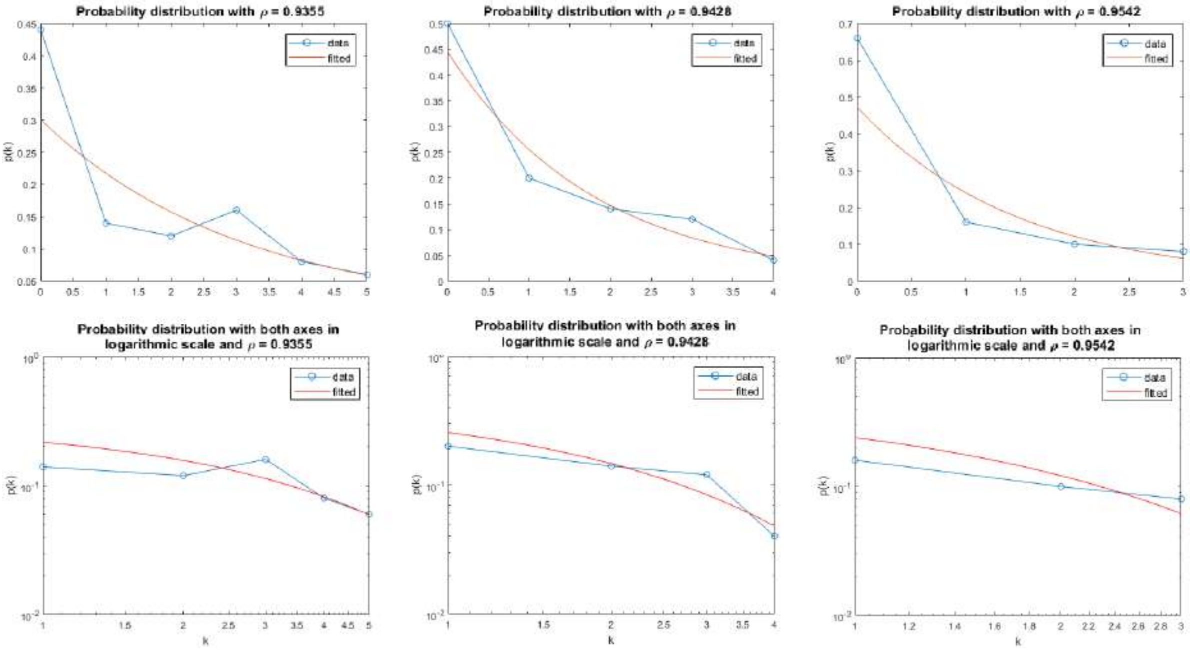

FIGURE 6: *Plot of the probability distribution for the three new subnetworks, both in usual and log-log scale*

	$\rho = 0.9355$	$\rho = 0.9428$	$\rho = 0.9542$
α	0.3005	0.4453	0.4728
γ	0.3244	0.5562	0.6801
$\varepsilon_{fitting}$	0.3030	0.1606	0.3066

TABLE 3: *Fitted power-law parameters for the three analyzed subnetworks*

by the market capitalization at the base period t_0, 3 September 2019 in this case, *i.e.*:

$$I_t = \frac{\sum_{i=1}^{N} p_{it}\, q_{it}}{\sum_{i=1}^{N} p_{it_0}\, q_{it_0}}, \tag{7}$$

where N are the few stocks selected with the previous technique, p_{it} and p_{it_0} are the prices of the stock i at time t and t_0 while q_{it} and q_{it_0} are its number of outstanding shares at the same times. With this simple weighted index formulation, it is given more importance to the companies that have a greater amount of outstanding shares: it makes perfectly sense because, in this way, the index reflects how the market really acts since the largest companies have a bigger impact on the market than the smallest ones.

To apply this formula, a meticulous work has been done in order to reconstruct the number of outstanding shares for each time of the whole period from 3 September 2019 to 31 August 2021. The current amount of outstanding shares and the capital changes happened in a selected period are available on Thomson Reuters so, proceeding backwards, the searched quantity has been reconstructed for each time. For more safety, this information has been also searched on another site (*MacroTrends website*) where the amount of outstanding shares is reported periodically, mostly quarterly: a change in this number has been considered only if substantial and not of small entity. By crossing the results collected from the two sites, it has been obtained that the only company with an important capital change in the considered period is SIEGn.DE. More precisely, on 29 September 2020 the corporation was subjected to a demerger, bringing to a halving of the amount of its outstanding shares. It can be interesting to list the names of the most important stocks, sorted with respect to the number of their outstanding shares at time t_0, for each of the three selected values of the threshold ρ, since these are the most influential in the index computation. For $\rho =$ 0.9355 the first five stocks are, in descending order of importance, ISP.MI, SAN.MC, BBVA.MC,

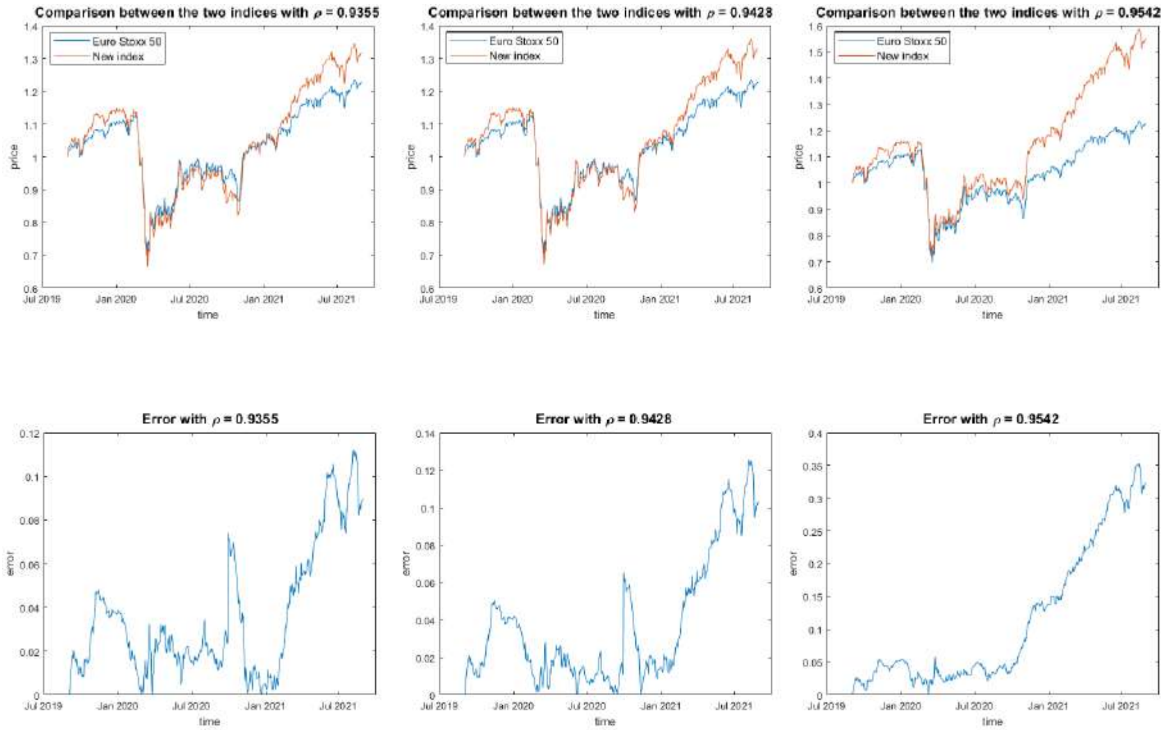

FIGURE 7: Plot of the Euro Stoxx 50 against the new index daily closing prices for the period from 3 September 2019 to 31 August 2021, with the corresponding error, for the three selected ρ

TEF.MC & INGA.AS. With the reduction of the number of companies given by the increase of ρ, BBVA.MC disappears from the top five of ρ = 0.9428 and ρ = 0.9542 which both include ISP.MI, SAN.MC, TEF.MC, INGA.AS & ENI.MI. It can be noticed that the top five is always composed of a majority of companies engaged in the banking sector, which nowadays is one of the most important. Therefore, for each of the three selected values of the threshold ρ, the new index has been computed.

The final step is to check if the obtained daily closing prices fit well against the Euro Stoxx 50 ones. Since the formula used in order to compute the new index, Equation (7), is a market capitalization formulation normalized by the market capitalization at the base period t_0, in order to be able to compare the two price evolutions a normalization of the Euro Stoxx 50 prices with the one at time t_0 has also been made. To get a more precise idea of the quality of the fitting, it is also important to graph the error which is simply given by the absolute value of the difference between the two interested quantities at each time and so:

$$Error_t = \left| \frac{p_t}{p_{t_0}} - I_t \right|, \tag{8}$$

where p_t and p_{t_0} are the Euro Stoxx 50 daily closing prices at time t and t_0, respectively, while I_t is the new index price at time t. Obviously, also in the computation of the error the Euro Stoxx 50 prices have been normalized. In this way the normalized prices for the three selected threshold values can be compared in Figure 7 where also the error evolution is illustrated.

As expected, the error increases with the growth of ρ because less and less companies are considered in the new index calculation. In the first case the error remains under 0.12 and twenty eight out of fifty stocks are taken into account but the situation is getting worse for the other two cases: the selected corporations become twenty five and seventeen, bringing the error to reach values near 0.13 and 0.35 for ρ = 0.9428 and ρ = 0.9542 respectively. Despite this, it is interesting to notice that the trend of the fitted prices is similar in all the cases.

Doubtless, the first thing that catches the eye from all the graphs is that the new index is a good fitting for the Euro Stoxx 50 prices for just over the first half of the period: the red curve differs most from the blue one from around the end of 2020 and the diversity continues to increase over time. This discrepancy is well captured by the error graphs which are, in all the analyzed cases but in particular in the last one, an increasing function. More precisely, they show a quite stable trend until the end of 2020 when they begin to rise for the remaining period.

A possible cause of this behaviour could lie in the fact that, by hypothesis, the composition of the new index has been kept constant throughout the whole period while, as it is well known, the companies included in the Euro Stoxx 50 computation are revisited annually. In fact, by controlling the variations in the composition between 3 September 2019 and 31 August 2021, it can be seen that, excluding a single change happened on 23 September 2019 when Unibail-Rodamco-Westfield SE was excluded and Deutsche Böerse AG was inserted, all the other ones occurred from 21 September 2020 onwards. In particular, in this last date there was a great change of the initial configuration because five companies have been replaced: Telefónica S.A., Société Générale S.A., Orange S.A., Fresenius SE & Co.KGaA and Banco Bilbao Vizcaya Argentaria S.A. gave way to Vonovia SE, Prosus N.V., Pernod Ricard S.A., Kone Oyj and Adyen N.V. On 2020 another variation happened on 30 November, when Unilever PLC was replaced by Flutter Entertainment plc, while during 2021 the last change occurred on 22 March, day in which Infineon Technologies AG substituted Nokia Oyj. Summing up, the considered configuration at time t_0 is different from the final composition of the Euro Stoxx 50 of eight out of fifty companies, an amount that can not be neglected. By comparing the corporations eliminated from the initial composition of the Euro Stoxx 50 with those present in the analyzed subnetworks, it comes to light that for $\rho = 0.9355$ five of them are included in the new index computation while four and three are involved in the latter two cases, for $\rho = 0.9428$ and $\rho = 0.9542$ respectively. From this observation it would seem natural to have a greater discrepancy between the two curves in the first case, in which the number of different stocks that make up the two compositions is greater, but this does not happen. Therefore the cause of this particular behaviour of the fitted curve must be sought elsewhere.

To support this deduction, the new index using all the fifty companies involved at time t_0 has also been computed and compared to the one calculated with less stocks in order to keep the composition constant over time in both cases. The plots obtained are similar to those previously analyzed: the new index computed with less stocks is still a good fitting for only the first part of the considered period and the error is an increasing function and it grows with the rise of the threshold value.

Another possible cause of this particular performance could lie in the price trends of the companies excluded from the calculation of the new index. To follow this trail, first of all the fifty corporations have been sorted in decreasing order with respect to their number of outstanding share at t_0 so as to have in first position the company with the greatest weight in the calculation of the new index. Then, this list of names has been compared to the stocks present in each of the three constructed subnetwork and the price evolutions of the companies not present have been plotted. From this analysis it would seem that, especially in the last case, the increase of the error in the final stretch is due to the continuous elimination of companies which have an opposite trend with respect to the one of the price itself: since they act as a balance for the final trend, they are an important loss which influences the general behaviour of the price leading to its considerable overestimation.

3.4 Introduction of the ESG Score and Analysis

We are in an era in which climate issues are starting to enter the daily lives of all of us and, therefore, to be an evaluation parameter in not only personal financial choices. In fact, in recent years, sustainable finance is becoming ever more popular: sustainable stocks often perform better than classic ones and companies with higher ESG score, generally, obtain greater returns. Therefore, in this final part of the work it has been decided to consider the effect of the ESG score in the financial market including it in the computation of the similarity between stocks. In this way it is evaluated if, jointly considering the price trend and the ESG score of the companies, some significant interconnections between the corporations, that would not be highlighted simply considering the price evolution, are captured.

In practice, the ESG score of each of the fifty selected companies has been collected by Thomson Reuters, one of the main ESG rating providers. This corporation elaborates the ESG score according to the three traditional pillars (Environmental, Social and Governance) each of one based on different categories of analysis, 10 in total: after collecting the data concerning each category, often based on information published in the press or otherwise in the public domain, by using an internal model it aggregates all the results into a single numerical score, with value between 0 and 100, which is then transformed into a letter, in the range [D–,A+], to be included in the technical reports [12]. Data in hand, looking at the average values of the ESG scores of each company in detail, it can be seen that the lowest value is 58.6519 and, even more interesting, 80% of the stocks have an average ESG score above 75, a clear signal that the selected corporations care about sustainable issues.

Once the ESG scores have been selected for all the fifty companies involved in the Euro Stoxx 50 at t_0 and for the period between 3 September 2019 to 31 August 2021, a new method in order to define the similarity between different corporations has been introduced. In the previous section two companies were considered similar if their prices moved in an analogous way over the studied period while, now, the factors to be taken into account for the similarity construction are two, prices and ESG scores. In order to combine them into a unique variable to be analyzed, a composite indicator has been created [6].

The first step in order to merge two individual factors into a single one consists in making them comparable thanks to a normalization method because they generally have different unit of measure. In this work, about the price normalization a mixed method has been adopted: focusing on each stock, for each time t between 3 September 2019 and 31 August 2021 its prices have been normalized by the maximum one obtained over the whole selected period, so:

$$p_j(t) = \frac{x_{1j}(t)}{\max_t x_{1j}(t)}, \tag{9}$$

where $x_{1j}(t)$ is the price of the stock j at time t. In this way the normalized prices are in the range (0,1] where the upper bound is in correspondence of the highest price of the whole time series.

Regarding the ESG scores, looking at all the values for each stock and for each time, the maximum and the minimum are, respectively, 94.10 and 55.92. Therefore, in order to create a normalization that takes into account only the values in this range, a linear interpolation has been used: a weight of 0.1 has been given to the smallest value, not 0 to have no null values in the correlation matrix, and the maximum weight of 1 at the greatest one. So the formula used to compute the normalized ESG score is:

$$esg_j(t) = 1 - \frac{\max_{j,t} x_{2j}(t) - x_{2j}(t)}{\max_{j,t} x_{2j}(t) - \min_{j,t} x_{2j}(t)} \cdot 0.9, \tag{10}$$

where $x_{2j}(t)$ is the ESG score of the stock j at time t.

In the final step the two individual indicators have been aggregated in a unique one: in order to impose an interaction between them, a geometric mean has been adopted as combination method:

$$CI_j(t) = \sqrt{p_j(t) \cdot esg_j(t)}, \tag{11}$$

where $CI_j(t)$ is the composite indicator at time t of the stock j while $p_j(t)$ and $esg_j(t)$ the normalized price and ESG score of the stock j at time t.

At this point, all the previous analysis made using only the prices has been redone with this composite indicator as a starting point: a correlation matrix has been computed in order to construct a scale-free network where the links express the similarity between the nodes, only some stocks have been selected in order to create a new index used to approximate the Euro Stoxx 50 prices and, finally, the results are compared to the ones of the previous part in order to understand if the introduction of the ESG scores have been led to an improvement in the index calculation.

The starting point for all the following analysis, as already seen, is the creation of the correlation matrix which has been computed with the composite indicator and it is illustrated in Figure 8.

By comparing it with the old one in Figure 2, some little changes can be visualized. First of all, in general this time the similarity between the stocks got a little worse since the amount of correlation coefficients above 0.5 is decreased to 49.71%. In addition to SAPG.DE & SASY.PA, even LINI.DE has almost no correlation with all the other stocks, except for SAPG.DE with which it has a good one. Two new groups of medium-high correlation appear: SCHN.PA, ABI.BR, DTEGn.DE & DAIGn.DE with an average of 0.8755 and CRH.I, AD.AS & VIV.PA with a mean of 0.8083. Some of the previous sets remain the same while others lose some elements, namely from the big group composed of AXAF.PA, ADSGn.DE, SAN.MC, BMWG.DE, SGEF.PA, SAF.PA, ISP.MI & INGA.AS only the last three remain well correlated each other while FREG.DE & ORAN.PA are excluded from the group of SOGN.PA, TEF.MC & URW.AS. Other two evident changes happens for AD.AS, which passes

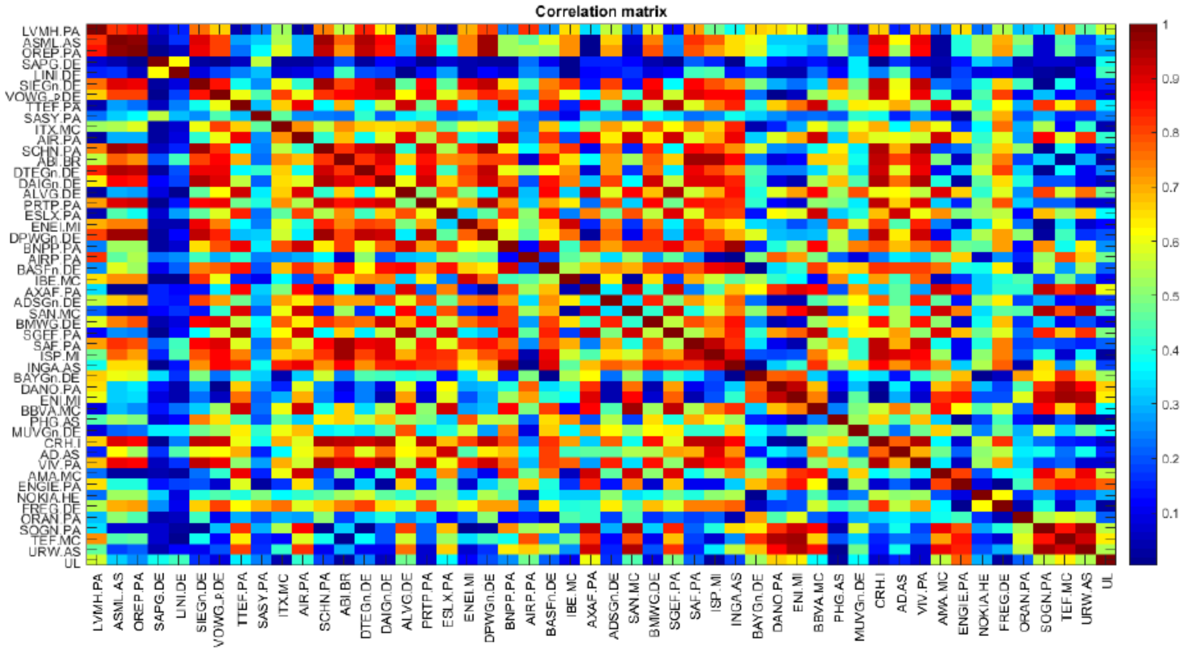

FIGURE 8: Correlation matrix for the composite indicator of selected stocks from 3 September 2019 to 31 August 2021

	$\rho = 0$	$\rho = 0.3383$	$\rho = 0.5771$	$\rho = 0.8557$	$\rho = 0.9353$	$\rho = 0.9751$
N	50	50	50	50	50	50
L	1225	806	514	139	33	0
d	1	0.6580	0.4196	0.1135	0.0269	0
k	49	32.24	20.56	5.56	1.32	0
h	1	1.0651	1.1821	1.6769	2.5023	NaN
D	1	2	4	8	6	0
avg_l	1	1.3420	1.3214	2.8319	2.1759	NaN
C	1	0.7726	0.7792	0.6766	0.4912	NaN

TABLE 4: Network parameters for the six analyzed graphs with the introduction of the ESG score

from being little correlated with all the other stocks to having a good correlation with most of them, and for ORAN.PA, for which exactly the opposite occurs. Another important change resulting from the ESG scores introduction is that there are more stocks negatively correlated each other. In fact, this time, the number of stocks with no negative correlation coefficients decreases to two, SASY.PA & MUVGn.DE, while the one with the bigger number of negative correlation values becomes LINI.DE with a greatly increased count, 32 cases out of 49 against the 22 of the previous case.

From a general look on the new networks constructed varying the threshold ρ using the same six values of the parameter of the previous situation, the most evident difference from the old graphs happens for the last case ($\rho = 0.9751$) in which there are no links because the biggest value of the correlation matrix has dropped to 0.9705. By comparing the new network parameters reported in Table 4 with the old ones in Table 2, it is possible to notice that there are no significant alterations in their general trend:

as expected, since overall the correlation matrix has lower coefficients than before, the number of links L, the density d and the average degree k are a little lower than before.

In order to proceed with the analysis, it has been found that the scale-free interval for this new situation is for ρ about in [0.9417, 0.9664]. Since this range does not completely contain the old one,

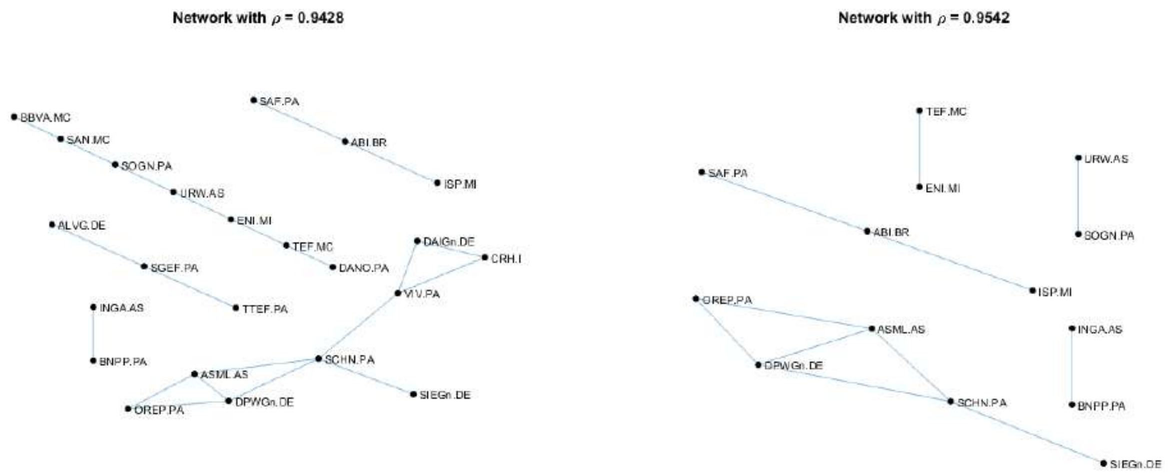

FIGURE 9: Graph representation for the two new subnetworks obtained with the introduction of the ESG score

it is possible to make an analogy between the Euro Stoxx 50 approximation with less stocks with and without the ESG score only for two threshold values: the new subnetworks constructed with ρ = 0.9428 and ρ = 0.9542 are reported in Figure 9.

By comparing them with the ones in Figure 5, as expected there are no radical changes: the components present little variations in their composition, both in nodes and links, but nothing stricking. It can be seen that for ρ = 0.9428 ORAN.PA, AMA.MC, LINI.DE, LVMH.PA, AIR.PA, BMWG.DE & AXAF.PA have been excluded and replaced by BBVA.MC, ABI.BR, CRH.I, VIV.PA & SGEF.PA while for ρ = 0.9542 ABI.BR, SAF.PA & SIEGn.DE entered in the network instead of BMWG.DE, DAIGn.DE, SAN.MC, TTEF.PA, LVMH.PA & LINI.DE.

Subsequently, exactly as before, a new index has been computed using the few stocks of these new subnetworks in order to approximate the Euro Stoxx 50 daily closing price evolution. Since the new index calculated using all the fifty companies involved is a good substitute of the Euro Stoxx 50, the subsequent analyzes have been made using it as a reference index to have more meaningful results. The normalized prices for the two selected values of the threshold using all the fifty stocks and only a few ones are reported in Figure 10.

As can be noticed, the two cases are very different: for ρ = 0.9428 the fit is not very good for the central part of the considered period, where the error takes very high values compared to the ones of the other dates, while for ρ = 0.9542 it has the usual trend in which the two curves differ more on the last months giving rise to an increasing error over time. To better understand the nature of the evolution of the error obtained in the attempt to approximate the index with fewer companies, similarly to what was done earlier for the study without the inclusion of the ESG scores, the price trends of the stocks involved in the reduced index has been examined for both the considered cases. Regarding the case with ρ = 0.9428, in the central part corresponding approximately to 2020, most companies, more precisely fourteen out of twenty three, show a downward trend with respect to the reference index curve so that the obtained price evolution is an underestimate. Instead, as regards ρ = 0.9542, in the last stretch seven out of fourteen stocks perform much higher than the reference index price curve, even reaching value 3.5 in the most striking case, so that the final result is an increasing overestimate of the Euro Stoxx 50 price.

The next and most important step of the whole work is to check if the inclusion of the ESG scores in the index calculation has led to some improvements in its performance. By comparing Figures 7 and 10, it would seem that there is a general better fitting in the second case. In fact, the error is more contained when the ESG scores are also taken into account: in particular for ρ = 0.9428 it stays below the value 0.1 instead of 0.13 and, even more, for ρ = 0.9542 it goes from a maximum near 0.35 to 0.15. To support this visual result with a mathematical check, a more in-depth error analysis has been made: the mean and the variance of the absolute and relative errors have been computed and the obtained results are reported in Table 5.

As expected it can be seen that, in both cases, on average the error diminishes when the ESG scores are included in the analysis, both as regards the absolute and the relative errors. Moreover,

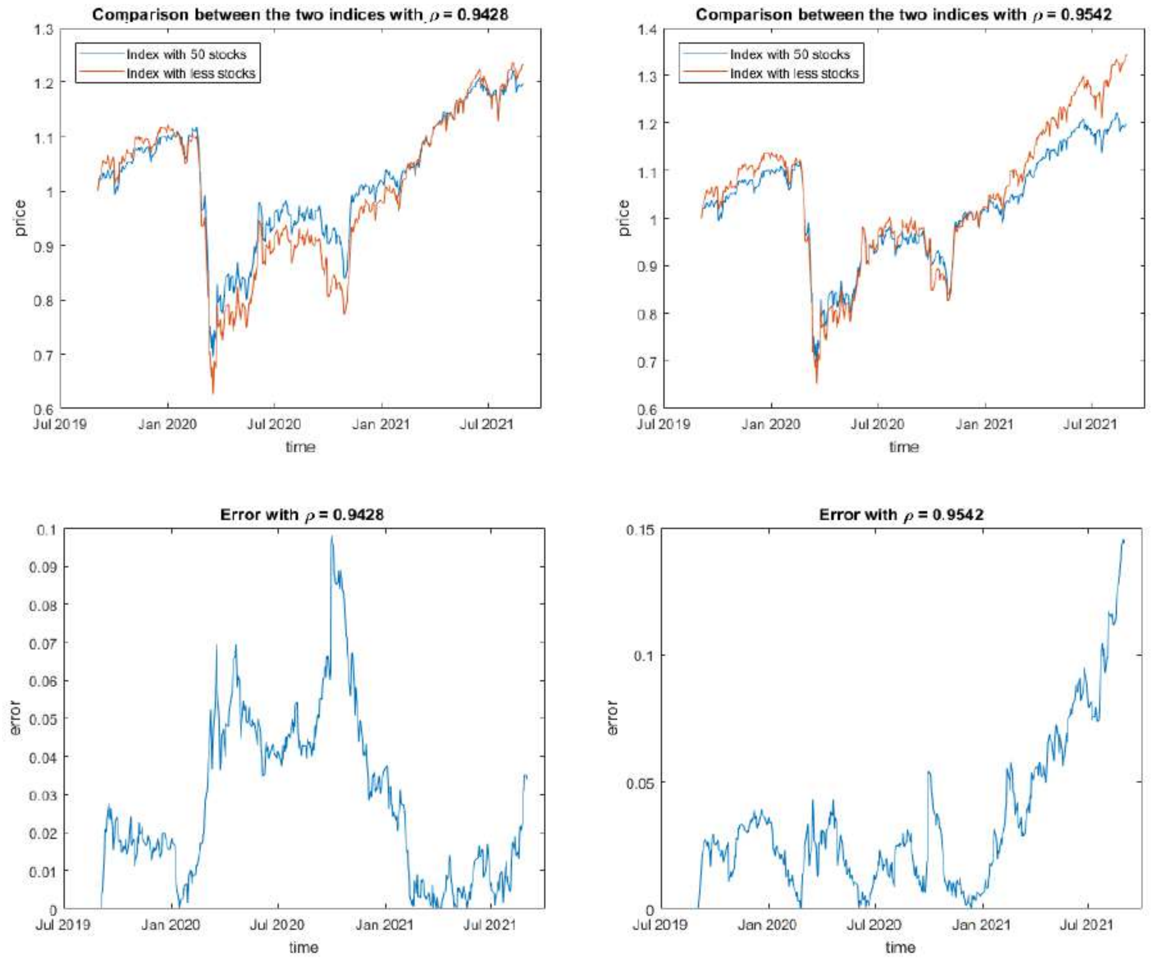

FIGURE 10: *Plot of the new index daily closing prices considering all the fifty stocks and only few ones with the introduction of the ESG score for the period from 3 September 2019 to 31 August 2021, with the corresponding error, for the two selected ρ*

it can be noticed that the variance has a considerable decrease passing from the initial situation to the second one, indicating that in the new scenario the error is less volatile and so that the approximation is more stable and precise.

Notice that, even if for both the chosen ρ values the number of stocks used in the construction of the new index is less when the ESG scores are added, more precisely for $\rho = 0.9428$ there are 2 less companies and for $\rho = 0.9542$ these are 3, the final fitting is better. This is a very interesting result because, as it happens in general, inserting more indicators in the analysis allows to capture more significant interconnections between the companies so that it is possible to have a better description of the financial market even using fewer players.

Finally, to better understand the reason behind the fitting improvement, it might be interesting to focus on the companies that differ in the composition of the reduced index between the case in which the similarity is based only on the daily closing price time series and the one where the ESG scores also come into play.

As usual, the fitting depends a lot on the prices of the selected companies and the results are in line with the expectations. As far as the ESG scores are concerned, the analysis shows a less intuitive and obvious result: it is not true that a company with a higher score better describes the trend of the index. In fact, for both the chosen ρ values, the corporations introduced in the basket used to reproduce the Euro Stoxx 50 price with the addition of the ESG score have a lower scoring than those they replaced. This is because the stocks that best replicate the index are those that are most correlated to each other, *i.e.* those that have the most similar and not higher ESG score.

			Threshold Value		
			$\rho = 0.9355$	$\rho = 0.9428$	$\rho = 0.9542$
Without ESG	Absolute Error	Mean	0.0430	0.0453	0.1276
		Variance	0.0013	0.0017	0.0124
	Relative Error	Mean	0.0396	0.0412	0.1180
		Variance	$8.8095\text{x}10^{-4}$	0.0012	0.0088
With ESG	Absolute Error	Mean	/	0.0289	0.0350
		Variance	/	$4.9067\text{x}10^{-4}$	$8.9536\text{x}10^{-4}$
	Relative Error	Mean	/	0.0306	0.0328
		Variance	/	$6.6305\text{x}10^{-4}$	$6.1326\text{x}10^{-4}$

TABLE 5: Mean and variance of the absolute and relative errors for all the analyzed cases, both considering only the daily closing prices and then adding the ESG score

4. Conclusions

In the end, after all these analyzes, it is possible to conclude that a good methodology to construct a new index that behaves like the Euro Stoxx 50, but which considers a smaller number of stocks, can be to choose the cluster of companies that are most correlated each other, both in a positive or in a negative way. Indeed, as seen, this relatively small subset of stocks contained in the basket is equally able to effectively capture and describe the trend of the studied index.

Furthermore, the study shows that it is even better to include the ESG score of each company in the selection criterion: when both the individual indicators are merged, the new created index can be described by fewer companies and, besides, it has a trend closer to the one of the reference index and so a better final fitting has been found. In this way it has been verified that the ESG score has an effective influence on the financial market since some significant interconnections between the corporations, that would not be highlighted simply considering the price evolution, are captured.

Moreover, it is possible to deduce that it is not necessary to have a high ESG score in order to be an influential company in the market. Certainly nowadays having a high ESG score makes a company more attractive to customers and therefore leads to higher earnings, but this does not necessarily lead it to be a key company on the market. In fact, from this work it has been seen that some stocks added in the reduced basket, used to replicate the Euro Stoxx 50 when the ESG score has been introduced in the analysis, have a smaller ESG score with respect to the companies that have been removed with the selection process.

Finally, from the results it is clear the key role of the threshold ρ in the selection process of the subgroup of the nodes used to replicate the Euro Stoxx 50 price. In fact, it is important to notice that varying its value very different fitting curves are obtained so a good strategy could be to choose a high ρ value, necessary condition to have a scale-free network, such that the final fitting is as accurate as possible over the whole analysis period.

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